Workshop 4.
Thinking Like a Mathematician

Towers of Hanoi is a game played by Buddhist monks. According to the rules of the game, when the last move of the game is performed, the world will end. In the sixth grade, students in the Kenilworth study try this game and look for a general formula to find the total number of moves. The steps they follow in solving the "mystery" closely parallel some strategies that professional mathematician Fern Hunt uses in her work.

Part 1—Strategies for Solving Problems

Our brief profile of mathematician Fern Hunt shows that collaboration is an essential part of doing mathematics in the real world. Fern uses the metaphor of a game (The Towers of Hanoi) to show the types of strategies that mathematicians might use in solving real-world problems. Under the guidance of the late Robert B. Davis, Kenilworth sixth-graders put into practice some of these strategies, working together enthusiastically on the challenge of uncovering the mystery of the Towers of Hanoi.

On-Screen Participants: Dr. Fern Hunt, National Institute for Science and Technology; and Dr. Robert B. Davis, Rutgers University. **Student Participants:** Brian, Jeff, Romina, Michael, Matt, Stephanie, Amy Lynn, Dana, Ankur, and other sixth-graders, Kenilworth Public Schools.

Part 2—Encouraging Students To Think

In Provo, Utah, Presidential-award-winning math teacher Janet Walter has just inherited a ninth-grade Algebra I class mid-year. She is trying to help her students be more confident discussing their developing mathematical ideas. In Kenilworth, Mike, a 10th-grade student in the long-term study, shares his insights. His ideas quickly evolve into a firmly established methodology that is used again and again to solve problems. Romina, another student in the study, responds to a challenge presented by Ankur, one of her classmates, and presents her solution to the group.

On-Screen Participants: Janet Walter, Provo, Utah Public Schools. **Student Participants**: Algebra I Class, Provo, Utah; Michael, Jeff, Romina, Ankur, and Brian, Kenilworth Public Schools.

On-Site Activities and Timeline

60 minutes

Getting Ready

Share and discuss solutions to Ankur's Challenge and Towers of Hanoi, your homework from Workshop 3.

60 minutes

Watch the Workshop Video

Part 1

On-Screen Math Activities

Towers of Hanoi
See pages 34-35.

Focus Question

We've seen students using a variety of problem-solving strategies to approach the Towers of Hanoi problem. What strategies have you observed your students using to solve difficult problems?

Part 2

On-Screen Math Activities

Scatter Plot (Provo, Utah)
The students' assignment was to plot a number of data points on a coordinate grid and then draw a line that best describes the data. They followed up by writing an equation for the line in the form: $y = ax + b$.

10th Grade: Revisiting the Pizza Problem
How many pizzas can you make by selecting from four different toppings?

Romina's Proof
How many different towers can you make four blocks high when selecting from three colors, if there is at least one of each color in each tower?

Focus Question

Is Romina's argument convincing? Why or why not?

On-Site Activities and Timeline

30 minutes

Going Further

Please address as many of these questions as possible during your allotted time and consider the remainder as part of your homework.

Episode Box One: Fern Hunt—Mathematician

Fern Hunt tells us her view on how mathematicians solve problems. For example, she points out that principles found in games and puzzles often engage the creative powers of the mind.

- Discuss this idea.

She indicated further that mathematicians, in dealing with complex problems, often simplify the situation in order to look at a related, but simpler, problem.

- How do you relate Fern Hunt's ideas to how you solve problems? To how the students you teach solve problems?

On-Site Activities and Timeline

Going Further, cont'd.

Episode Box Two: Sixth Grade—The Towers of Hanoi

Mathematician Fern Hunt used the Towers of Hanoi problem as an example of how she and other mathematicians solve problems. In the sixth grade, the students in the Kenilworth study worked on the same problem.

- Relate Fern Hunt's ideas about how mathematicians solve problems to the way the sixth-grade children work on the Towers of Hanoi problem in the video.

Michael explains the "doubling plus one" rule by moving stacks of discs of a particular height.

- Explain his idea. Is it a convincing argument for the rule? Why or why not?

The sixth-graders in this video clip are posing rules for explaining the Towers of Hanoi puzzle for stacks of discs *n* high. Matt suggests that algebra is being used when he offers that "Two *w* plus one equals the number you say."

- What evidence, if any, do you see of algebra or algebraic thinking in the explanations and work of these students? What is your view of the challenge given to these students for finding a general solution? What mathematical ideas are the students using in the pursuit of the solution?

- Comment on the table produced by the students for recording their data.

□	△	
1	1	
		2
2	3	
		4
3	7	
		8
4	15	
		16
5	31	
		32
6	63	
		64
7	127	

On-Site Activities and Timeline

Going Further, cont'd.

Episode Box Three: Ninth Grade—Algebraic Thinking
Janet was assigned a ninth-grade Algebra I class part-way through the year.

- Discuss Janet's way of working with the students. How engaged are they? What mathematical ideas are the students building?

Episode Box Four: 10th Grade—Michael's Binary Code
Michael is working with a group of students in an after-school group interview session.

- What mathematics is Michael doing?

Recall that Brandon recorded his solution to the four-topping Pizza problem with 0's and 1's and related this work to the Towers Four-High problem.

For example, Brandon, grade four, wrote:

1000
0100
0010 etc.

Recall, Michael's notation in grade 11:

0001
0010
0011 etc.

- How does Michael's binary code differ from Brandon's? Are there advantages or disadvantages to each system? Discuss the use of notation in the problem solving of these students.

Episode Box Five: 10th Grade—Romina's Proof
Ankur presented the following problem to the group: "Find as many towers as possible that are four cubes tall if you can select from three colors and there must be at least one of each color in each tower. How do you know that you have found all the possibilities? Build a solution, selecting from three colors of Unifix® cubes. Convince your peers that you have found all the possibilities, no more and no fewer."

On the next page is Romina's solution to Ankur's challenge.

- Discuss Romina's solution and its presentation to the group. Were you convinced? Why or why not?

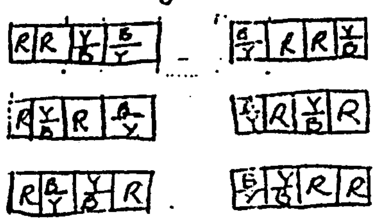
On-Site Activities and Timeline

Going Further, cont'd.

TOWER PROBLEM

HOW MANY TOWERS CAN YOU BUILD FOUR HIGH WITH THE CHOICE OF THREE COLORS AND HAVING ALL THREE COLORS IN EACH TOWER.

I first approached this problem realizing that if there are three different colors and four different areas where they can be put then there will be a double of one color, and two distinct colors, in each tower. I then tried the problem with the colors yellow (Y), Blue (B) and Red (R) with red being the duplicated color:



6 ← positions of color being dup
× 2 ← combo - BY or YB
12 ← total for red being doub
× 3 ← # of colors
36 ← # of combinations

I placed the two reds in every possible way they could be, which left me with 6 possibilities. In the remaining blocks there can only be a combination of two possibilities: yellow and blue OR blue and yellow. Since there are only two remaining possibilities of combinations one must multiply the 6 by 2 leaving 12. The 12 represents the number of possibilities of towers being built with R as the duplicated color. Since there are 3 colors, and each can be the doubled color, one must multiply the 12 by 3 indicating the rest, yellow and blue have been the duplicated color in each. Leaving one with the answer of 36; the number of all possible combinations.

Romina's Solution

For Next Time

Homework Assignment

Preparing for Workshop 5

1. Study the Episode Boxes on the previous pages. Reflect on the questions and record your reactions and ideas in your journal.

2. Solve the World Series problem. In a World Series, two teams play each other in at least four and at most seven games. The first team to win four games is the winner of the World Series. Assuming that both teams are equally matched, what is the probability that a World Series will be won (a) in four games? (b) in five games? (c) in six games? (d) in seven games?

Notes