

Workshop 1.

Following Children’s Ideas in Mathematics

How far can children’s natural curiosity and interest in math take them? How do students do mathematics before they are taught procedures for solving problems? Dr. Carolyn Maher and a team of researchers from Rutgers University are pioneers in the effort to answer these questions. Their work documents how students approached a progressive series of challenging mathematics problems beginning in the first grade and continuing all the way through high school. This unprecedented study provides compelling evidence that, given the right conditions, students can accomplish amazing things in mathematics. In Workshop 1, we look at an overview of Dr. Maher’s work and its impact on a group of students from Kenilworth, New Jersey.

Part 1—The Youngest Mathematicians

Looking through the lens of developmental psychologist Dr. Herbert Ginsburg, what looks like play in preschool children is actually the beginning of serious mathematical thinking. Elementary school classrooms in Kenilworth provide a starting point for the Rutgers researchers to track how mathematical thinking develops as it begins to be formally introduced. We hear how Dr. Maher’s professional development work with teachers grew into a long-term study and we observe small groups of children begin to tackle a combinations problem called Shirts and Pants. Students then progress to a more complicated problem: Cups, Bowls, and Plates. Archival footage shows how the students worked toward a solution over time by devising their own way of representing and solving the problem.

On-Screen Participants: Dr. Herbert Ginsburg, Columbia University; Dr. Carolyn Maher, Rutgers University; Dr. Amy Martino, Rutgers University. **Student Participants:** Dana, Michael, and Stephanie, Kenilworth, New Jersey Public Schools, Grades 2 and 3.

Part 2—From “Towers” to High School

Continuing a chronological history of the long-term study, Dr. Maher introduces the Towers problem. The activity, which investigates how many different five-cube-high towers can be made by selecting from blocks of two colors, would be at home in any high school or college probability class. The difference is that Dr. Maher gives the students objects, not formulas, and asks them to justify their results. Students jump on the problem with enthusiasm, as the researchers purposefully step back to “see what happens.” As the study continues, the students take on more complex problems. However, during this time, the students in the focus group are also being affected by the school and community environment. In the ninth grade, the local school is closed, and the students are assigned to a geometry class where rote learning and memorization are the order of the day.

On-Screen Participants: Dr. Carolyn Maher, Rutgers University; Dr. Amy Martino, Rutgers University. **Student Participants:** Brian, Dana, Jeff, Michael, Michelle, Romina, and Stephanie, Kenilworth, New Jersey Public Schools, Grade 4.

On-Site Activities and Timeline

60 minutes

Getting Ready

Try the following activities for yourself in preparation for watching the workshop video.

1. Shirts and Pants

- a. Solve the problem below for yourself and develop a way to convince others that your solution is correct.

Stephen has a white shirt, a blue shirt, and a yellow shirt. He has a pair of blue jeans and a pair of white jeans. How many different outfits can he make?

- b. Share and compare your solution and justification with others in your group.
- c. Imagine giving this problem to students that you teach. Do you think it is an appropriate problem for your students? How do you think your students would solve and justify the problem?
- d. Solve the following extensions to the Shirts and Pants problem. Share your solutions.

1. Adding an Item

Remember that Stephen has a white shirt, a blue shirt, and a yellow shirt. He has a pair of blue jeans and a pair of white jeans. Stephen also has a brown belt and a black belt. How many different outfits can he make now?

2. Reversibility

Mario has exactly 16 different outfits. Decide how this might be possible. Specify what pieces of clothing he might have.

2. Cups, Bowls, and Plates

- a. Solve the following problem.

Pretend that there is a birthday party in your class today. It's your job to set the places with cups, bowls, and plates. The cups and bowls are blue or yellow. The plates are either blue, yellow, or orange. Is it possible for 10 children at the party to each have a different combination of cup, bowl, and plate?

Is it possible for 15 children at the party each to have a different combination of cup, bowl, and plate?

- b. Compare and justify your solutions with the group.
- c. Reflect on whether this is a problem you might use with your students. How do you think that your students would solve this problem?

On-Site Activities and Timeline

60 minutes

Watch the Workshop Video

Part 1

On-Screen Math Activities

Shirts and Pants

Stephen has a white shirt, a blue shirt, and a yellow shirt. He has a pair of blue jeans and a pair of white jeans. How many different outfits can Stephen make?

Cups, Bowls, and Plates

There is to be a birthday party for one of the students in class. There are blue and yellow cups, blue and yellow bowls, and blue, yellow, and orange plates. Can 10 children have a different combination of cups, bowls, and plates?

Focus Question

We've just seen the teacher/researchers repeating the same, or similar, problems three times over the course of almost a year. What can we say about the changes in the students' methods over time?

Part 2

On-Screen Math Activities

Towers

Build all possible towers that are five (or four, or three, or n) cubes high by selecting from plastic cubes in two colors. Provide a convincing argument that all possible arrangements have been found.

Focus Question

We've had a chance to look at an overview of the Rutgers' long-term study. In what ways are the students in the study similar to your students? How are they different?

On-Site Activities and Timeline

30 minutes

Going Further

Please address as many of these questions as possible during your allotted time and consider the remainder as part of your homework.

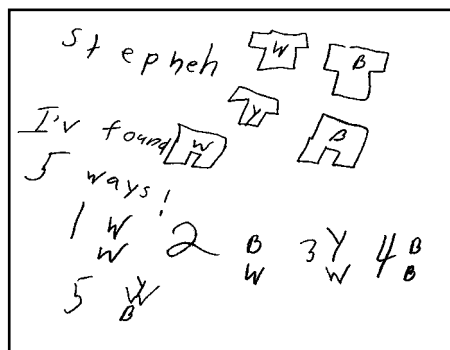
Episode Box One: Mathematical Thinking Among the Preschool Children

Professor Herbert Ginsburg described the activity of preschool children to be mathematical when it involved ideas like numbers, size comparison, measurement, and patterns.

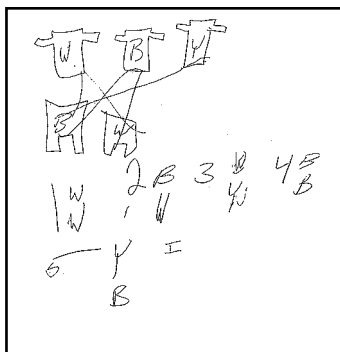
- What mathematics did you see the preschool children doing?

Episode Box Two: Shirts and Pants in Grade 2

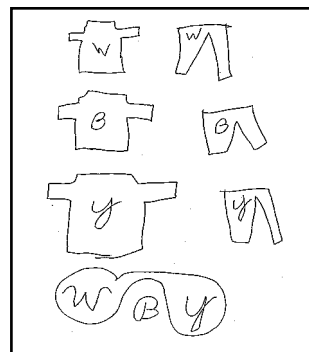
Stephanie, Dana, and Michael each solved the Shirts and Pants problem. They had different solutions. Although Stephanie and Dana both had 5 outfits, they were different. Michael's solution had 3 outfits, including his own pair of yellow pants.



Stephanie's Solution



Dana's Solution



Michael's Solution

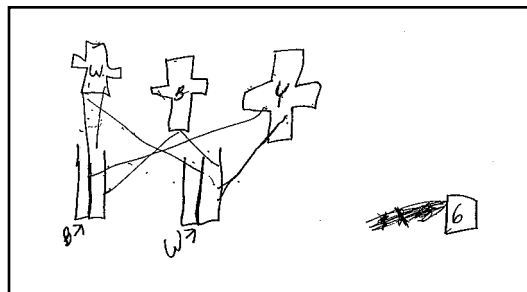
- Discuss the children's problem solving.

On-Site Activities and Timeline

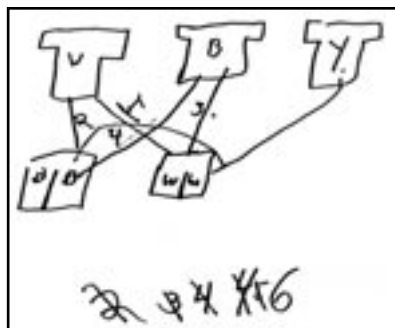
Going Further, cont'd.

Episode Box Three: Shirts and Pants in Grade 3

As third-graders, Dana, Stephanie, and Michael again solved the problem. There had been no intervention between the two problem sessions. This time, each child found 6 outfits as their solution.



Dana's Solution



Stephanie's Solution

Stephen has a white shirt, a blue shirt and a yellow shirt.
 He has a pair of blue jeans and a pair of white jeans.
 How many different outfits can he make?

1 W B B Y 4 W B Y
 2 W B B B B B B W

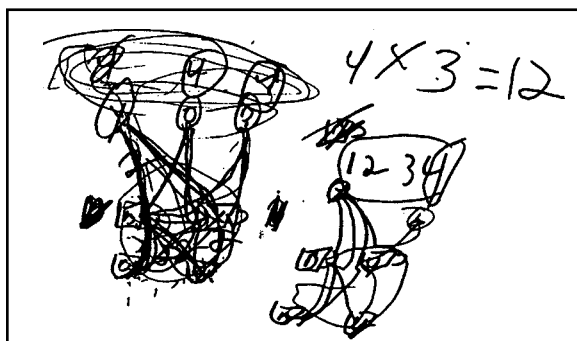
6
 Pairs

Michael's Solution

- Discuss the differences and/or similarities in the children's problem solving from grade two to grade three.

Episode Box Four: Cups, Bowls, and Plates

Stephanie and Dana solved the problem together. In a follow-up interview, Stephanie explained her solution using a diagram with lines connecting the three groups of items to find 12 combinations.



- Discuss the girls' "connecting tree" strategy.
- How might this strategy be related to the approach used in the Shirts and Pants activity?

For Next Time

Homework Assignment

Preparing for Workshop 2

1. Study the Episode Boxes on the previous pages. Reflect on the questions and record your reactions and ideas in your journal.
2. Try the Towers Four-High problem. You have two colors of Unifix® Cubes available with which to build towers. Your homework task is to make as many different-looking towers as possible, each exactly four cubes high. A tower always points up, with the little knob on top. Find a way to convince yourself and others that you have found all possible towers four cubes high and that you have no duplicates. Keep a record of your solution, including your justification, to share during Workshop 2.

After you have completed your solution for towers four cubes high, predict (without building the towers) the number of possible towers: (a) three cubes high and (b) five cubes high. Why do you think your predictions are correct?

Note: If you do not have Unifix® Cubes, cut out and use the “dark” and “light” cubes on the following pages.

3. Use any of the problems and extensions from Getting Ready of Workshop 1 with one or more of your students. Take notes of what the children do; reflect—in writing—about your own role in the activity; collect and bring all of the children’s written work to the next session to share.

Reading Assignment

The reading assignment can be found in the Appendix of this guide.

Davis, R. B., Maher, C. A., and Martino, A. “Using Videotapes To Study the Construction of Mathematical Knowledge of Individual Children Working in Groups.” *Journal of Science, Education and Technology*, 1(3), 177-189, 1992.

cubes

cubes

Notes
