REVOLUTION IN ONE CLASSROOM

(or, then again, was it?)

BY DAVID K. COHEN

AS MRS. Oublier sees it, her classroom is a new world. When she began six or seven years ago, she was a thoroughly traditional teacher. She reported that she followed the mathematics text. Her second graders spent most of their time on worksheets. Learning math meant memorizing facts and procedures. Then Mrs. O found a new way to teach math. The summer after her first year of teaching, she took a workshop in which she learned to focus lessons on students' understanding of mathematical ideas. She found ways to relate mathematical concepts to students' knowledge and experience. And she learned how to engage students in actively understanding mathematics.

Mrs. O's story is a timely one. I encountered her in the late 1980s, as reformers once again began trying to change mathematics teaching and learning from mechanical drill and memorization to reasoning and understanding. Since the early twentieth century, mathematicians and math educators had intermittently insisted that students should learn to reason mathematically, to apply mathematical ideas to everyday situations, and to understand the conceptual basis of mathematics. But in the 1980s, state and national education leaders, chagrined about reports of weak school performance and worried about America's economic situation, gave new force to demands for reform.

These are revolutionary aspirations, at least judged by current classroom practice. But the new ambitions are being taken quite seriously. The National Council of Teachers of Mathematics has formulated an ambitious new set of standards for teaching and curriculum, which have received favorable attention in many quarters, including the secretary of education and the president. Several states are trying to realize the new ideas. For instance , since 1985, California's department of education has been pressing a remarkable program of reform in mathematics teaching and learning. The state issued a new curriculum framework. It then required publishers to re-orient math textbooks to conform more closely to new ideas about instruction. It also began to re-write the state testing program so that it assesses students' understanding and reasoning. And it has been offering workshops and other assistance to teachers.

Mrs. O teaches in California and sees her work as part of the changes that the state is trying to promote. Her story is engaging, and so is she. She is considerate of her students, eager for them to learn, energetic, and attractive. These qualities would stand out anywhere, but they seem particularly vivid in her school—a drab collection of one-story concrete buildings that sprawl over several

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acres. Though clean and well managed, her school has none of the familiar signs of classy education. It has no legacy of experimentation or progressive pedagogy, or even of heavy spending on education. Only a minority of children come from well-off families. Most have middling or modest incomes, and many are eligible for Chapter 1 assistance. A sizable minority are on welfare. The school district is situated in a dusty corner of southern California, where city migrants are turning a rural town into a suburb. New condominiums are sprouting up all over the community, but one still sees pick-up trucks with rifle racks in their rear windows. Like several of her colleagues, Mrs. O works in a covey of tacky, portable, prefab classrooms, trucked into the back of the schoolyard to absorb growing enrollments on the cheap.

Mrs. O's story seems even more unlikely when considered against the history of American educational reform. Great plans for educational change are familiar in that history, but so are reports of failed reform. John Dewey and others announced a revolution in pedagogy just as our century opened, but apparently it fizzled, for classrooms changed only a little (Cuban, 1984). That also seems to have been the fate of the earlier "new math" in the 1950s and 1960s and of related efforts to improve science teaching (Welch, 1979). Since then, many studies of instructional innovation have embroidered these old themes of great ambitions and modest results (Gross, et al., 1971; Berman and McLaughlin, 1977; Rowan and Guthrie, 1989; Cohen, 1988).

Some analysts attribute these results to teachers' resistance, saying that entrenched classroom habits defeat reform (Gross, et al., 1971). Others report that many innovations fail because they are so poorly adapted to the classroom that even teachers who avidly desire change can do little (Cuban, 1984; Cuban, 1986). Mrs O's revolution looks particularly appealing against this background. She eagerly embraced change, rather than resisting it, finding new ideas and materials that worked in her classroom. Mrs. O sees her class as a success for the new mathematics Framework. She reports that her math teaching has wound up where the Framework wants it to be.

SOMETHING OLD AND SOMETHING NEW

One prominent feature of Mrs. O's teaching is her use of innovative instructional materials and activities designed to help students make sense of mathematics. But she used these new activities and materials quite traditionally, as though mathematics contained only right and wrong answers. Similarly, while she had revised the class organization and activities to help students understand math, she managed the discourse in ways that discouraged exploration of students' understanding.

In fact, Mrs. O's lessons were quite mixed. They contained some important elements that reformers embraced, but others that they branded inadequate. Her classes present an extraordinary *mélange* of tradition and novel approaches to math instruction, which is one reason that they deserve attention. For such mixtures are quite common in instructional innovations, though little noticed. As teachers and students try to find their way from familiar practices to new ones, they cobble new

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deas together with old practices. Teachers' ingenuity is remarkable, but the mixtures raise fundamental questions. Can we say that an innovation has made much progress when it is tangled up with many traditional practices? What might it take to help teachers continue to learn and change? These questions have a special urgency just now, as reformers urge teachers to radically revise their work in math and other subjects.

New Materials, Old Mathematics

From one angle, the curriculum and instructional materials in this class looked just like what the new California math Framework invited. For instance, Mrs. O regularly asked her second graders to work on "number sentences." In one class that I observed, students had just done the problem: 10+4=14. Mrs. O then asked them to generate additional "number sentences" about 14. They volunteered various ways to write addition problems about fourteen—that is, 10+1+1+1+1=14, 5+5+4=14, etc. Some students proposed several ways to write subtraction problems—that is, 14-4=10, 14-10=4, etc. Most of the students' proposals were correct. Such work could make mathematical relationships more accessible, by coming at them with ordinary language rather than working only with bare numbers on a page. It also could unpack mathematical relationships, by offering different ways to get the same result. It could illuminate the reversible relations between addition and subtraction. And it could get students to do "mental math," i.e., to solve problems in their heads and thereby learn to see math as something to puzzle about and figure out, rather than just a bunch of facts and procedures to be memorized.

These are all things that the new Framework celebrated. It exhorted teachers to help students cultivate ". . . an attitude of curiosity and the willingness to probe and explore . . . " (California State Department of Education [CSDE], 1985, p.1). It also called for classroom work that helps students ". . . to understand why computational algorithms are constructed in particular forms . . ." (CSDE, 1985, p.4).

But Mrs. O conducted the entire exercise in a thoroughly traditional fashion. The class recited in response to the teacher's queries. Students' sentences were accepted if correct, and written down on the board. But they were turned down if incorrect, and not written on the board. Right answers were not explained, and wrong answers were treated as unreal. The Framework made no such distinction, arguing instead that understanding how to arrive at answers is an essential part of helping students figure out how mathematics works—no less important than whether the answers are right or wrong. The Framework criticized the usual algorithmic approach to mathematics, and the usual search for the right answer. It called for class discussion of problems as an important part of figuring out mathematical relationships (CSDE, 1985, pp. 13-14). But no one in Mrs. O's class was asked to explain his or her proposed number sentences, whether correct or not. No student was invited to demonstrate how he or she knew whether a sentence was correct or not. The teacher used a new mathematics curriculum, but used it in a way that conveyed a sense of mathematics as a fixed body of knowledge of right answers rather than as a field of inquiry.

The mixture of new mathematical ideas and materials with old mathematical knowledge and pedagogy showed up elsewhere in Mrs. O's work. She used concrete materials and other physical activities extensively to represent mathematical concepts in forms that are vivid and accessible to young children. She opened every day with a calendar activity in which she and the students gathered on a rug at one side of the room to count up the days of the school year. She used this activity for various purposes. During my first visit, she was familiarizing students with place value, regrouping, and odd and even numbers. As it happened, my visit began on the thirty-ninth day of the school year, and so the class counted to thirty-nine. They used single claps for most numbers but double claps for ten, twenty, etc. Thus, one physical activity represented the "tens" and distinguished them from another physical activity that was used to represent the "ones." The idea was that fundamental distinctions among types of numbers can be represented in ways that make immediate sense to young children and that will easily familiarize them with important mathematical ideas.

Mrs. O's class abounds with such activities and materials, and they are very different from the bare numbers on worksheets that would be found in a traditional math class. Her approach seems nicely attuned to the new Framework. For instance, that document argues that "many activities should involve concrete experiences so that students develop a sense of what numbers mean and how they are related before they are asked to add, subtract, multiply, or divide them (CSDE, 1985, p. 8). And it adds that "concrete materials provide a way for students to connect their own understandings about real objects and their own experiences to mathematical concepts. They gain direct experience with the underlying principles of each concept" (CSDE, 1985, p.15).

But it is one thing to embrace a doctrine of instruction and quite another to weave it deeply into one's practice. For even rather monotonous teaching comprises many different threads, and any new instructional element is somehow related to many others already there. The new thread can simply be dropped onto the fabric, and everything else left as is. Or new threads may be somehow woven into the fabric. Mrs. O introduced new threads but only slightly re-adjusted the old ones. Hence the novel materials and activities were infused with traditional messages about what mathematics was and what it meant to understand it.

These mixed qualities were vividly apparent in a lesson that focused on addition and subtraction with regrouping. The lesson occurred early in an eight-or ten-week cycle concerning these topics. Like many of Mrs. O's lessons, it combined a game-like activity with the use of concrete materials. She hoped to capture children's interest in math while helping them to understand it. Mrs. O introduced this lesson by announcing: "Boys and girls, today we are going to play a counting game. Inside this paper [holding up a wadded-up sheet of paper] is the secret message. . . ." Mrs. O unwadded the paper and held it up: "6" was inscribed. The number was important because it would establish the number base for the lesson: Six. In previous lessons, they had done the same thing with four and five. So part of the story here was exploring how things work in different number bases, and one reason for that, presumably, was to get some perspective on the base-ten system that we conventionally use. Mrs. O told the children that, as in the previous games, they would use a nonsense word in place of the secret number. This time they selected "cat's eye" to stand in for six.

With this groundwork laid, Mrs. O had "place-value boards" given to each student. She held up her board: It was roughly eight by eleven; one half was blue, the other white. She said: "We call this a place-value board. What do you notice about it?"

Cristie Smith, who turned out to be a steady infielder on Mrs. O's team, said: "There's a smiling face at the top." Mrs. O agreed, noting that the smiling face needed to be at the top at all times [that would keep the blue half of the board to everyone's left]. Several kids held theirs up for inspection from various angles, and she admonished them to leave the boards flat on their tables at all times.

"What else do we notice?" she inquired. Sam said that one half is blue and the other white. Mrs. O agreed and went on to say that ". . . the blue side will be the cat's eye side. During this game we will add one to the white side, and when we get a cat's eye, we will move it over to the blue side." With that, each student was given a small plastic tub, which contained a handful of dried beans and half a dozen small paper cups, perhaps a third the size of those dispensed in dentists' offices. This was the sum total of pre-lesson framing—no other discussion or description preceeded the work.

There was a small flurry of activity as students took their tubs and checked out the contents. Beans present nearly endless mischievous possibilities, and several of the kids seemed on the verge of exploring their properties as guided missiles. Mrs. O nipped off these investigations, saying: "Put your tubs at the top of your desks, and put both hands in the air." The students complied, as though in a small stagecoach robbery. "Please keep them up while I talk." She opened a spiral-bound book, not the school district's adopted text but *Math Their Way.* This was the innovative curriculum guide that had helped to spark her revolution. She looked at it from time to time as the lesson progressed but seemed to have quite a good grip on the activity.

Mrs. O got things off to a brisk start: "Boys and girls [who still were in the holdup], when I clap my hands, add a bean to the white side."

She clapped once vigorously, adding that they could put their hands down. "Now we are going to read what we have: What do we have?" (She led a choral chant of the answer.) "Zero cat's eye and one." She asked students to repeat that, and everyone did. She clapped again, and students obediently added a second bean to the white portion of the card. "What do we have now?" she inquired. Again she led a choral chant: "Zero cat's eye and two." So another part of the story in this lesson was place value: "Zero cat's eye denoted what would be the "tens" place in the base-ten numbering, and "two" is the "one's" place. Counting individual beans and beans grouped in "cat's eye" would give the kids a first-hand, physical sense of how place value worked in this and other number bases.

In these opening chants, as in all subsequent ones, Mrs. O performed as much as a drill sergeant as a choir director. Rather than establishing a beat and then maintaining it, she led each chant and the class followed at a split-second interval. Any kid who didn't grasp the idea needed only to wait for her cue or for his table-mates. Students were never invited or allowed to count on their own. Thus, while the *leitmotif* in their second chant was "zero cat's eye and two," there was an audible minor theme of "zero cat's eye and one." That several students repeated the first chant suggested that they did not get either the routine or its point.

Mrs. O moved right on nonetheless, saying that it ". . . is very important that you read the numbers with your hands." This was a matter to which she returned many times during the lesson, reminding children to put their little paws first on the beans in the white square and then on the little cups on the blue square as they incanted the mathematical chants. She seemed to feel it essential that they manipulate the concrete materials. Whenever she spotted a child who was not palpitating beans and cups, she walked over and moved their arms and hands for them.

Mrs. O led the bean adding and chants up to five. Then, with five beans down on everyone's card, she asked: "Now think ahead; when I clap my hands this time, what will you have on the white side?"

Reliable Cristie Smith scooped it up and threw smoothly to first: "Cat's eye."

Mrs. O led off again: "When you get a cat's eye, put all the beans in a paper cup and move them over." She clapped her hands for the cat's eye and then led the following chant: "Put the beans in the cup and move them over."

"Now let's read what we have." The chant rolled on: "One cat's eye and zero." A puzzling undercurrent of "one cat's eye and one" went unattended. She then led the class through a series of claps and chants, leading up to two cat's eyes. And then, with a methodical monotony, up to five cat's eyes and five. By the time they got to five cat's eyes and five, her claps had grown more perfunctory, and many kids had gotten the fidgets. But Mrs. O saw this chanting and bean-pawing as the high road to mathematical understanding and tenaciously drove her team on.

"Now, how many do we have?" "Five cat's eyes and five beans," came the chant. "Now we will take away one bean" (from the "ones" side of the board). "How many do we have?" Again the answering chant, again led by her, a fraction of a second earlier: "Five cat's eyes and four."

This was a crucial point, for the class was moving from a representation of addition with regrouping to a representation of subtraction with regrouping. It would have been an obvious moment for some such comment or discussion, at least if one saw the articulation of ideas as part of understanding mathematics. But Mrs. O did not comment or explain. She took an activity-based approach, as though all the important ideas were implicit, and better that way.

Thus the class counted down to five cat's eyes and zero. Mrs. O then asked, "What do we do now?" Jane responded: "Take a dish from the cat's eye side and move it to the white side." No explanation was requested or offered to embroider this response. Mrs. O simply approved the answer, clapped her hands, and everyone followed Jane's lead. With this, Mrs. O led the class back

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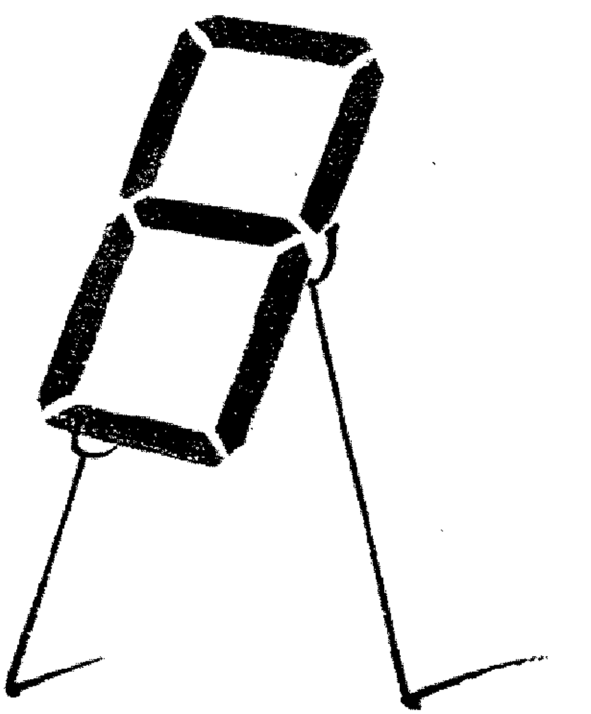

through each step, with claps, chants, and reminders to "read" the beans with their hands, down to zero cat's eye and zero beans. Everyone was flagging long before it was done, but not a chant was skipped or a movement missed.

Why did Mrs. O teach in this fashion? In an interview following the lesson, I asked what she thought the children learned from the exercise. She said that manipulating the materials helped them to understand what goes on in addition and subtraction with regrouping. She seemed convinced that these physical experiences caused learning, that mathematical knowledge arose from the activities.

Her inspiration for all this was *Math Their Way*, on which Mrs. O relied heavily. This increasingly popular book, a system of primary grade math teaching, announces that it will help "... to develop understanding and insight of the patterns of mathematics through the use of concrete materials" (Baratta-Lorton, 1976: xiv). Concrete materials and physical activities are crucial because they are believed to provide real experience with mathematics. The book sharply distinguishes between mathematical symbols and concepts and criticizes teaching with symbols. Symbols—that is, numbers— "... are not *the concept* [emphasis in original], they are only a representation of the concept, and as such are abstractions describing something which is not visible to the child. Real materials, on the other hand, can be manipulated to illustrate the concept concretely, and can be experienced visually by the child.... The emphasis throughout this book is making concepts, rather than numerical symbols, meaningful" (Baratta-Lorton, 1976: xiv).

Math Their Way fairly oozes with the belief that physical representations are much more real than symbolic ones. This idea is a recent mathematical mutation of the idea, at least as old as Rousseau, Pestalozzi, and James Fenimore Cooper, that experience is a better teacher than books. For experience is vivid, vital, and immediate, while books are all abstract ideas and dead formulations. *Math Their Way* also claims that concrete materials are developmentally desirable for young children. Numbers are referred to many times as an "adult" way of approaching math. That idea leads to another, still more important: If math is taught properly, it will be easy. Activities with concrete materials, the book insists, are the natural way for kids to learn math: "... if this foundation is firmly laid, dealing with abstract numbers will be effortless" (Baratta-Lorton, 1976: 176).

Stated so baldly, that seems a phenomenal claim. Simply beginning with the proper activities and materials ensures that math will be understood well and easily. But the idea is quite common. Pestalozzi might have cheered it. Many other pedagogical Romantics, Rousseau and Dewey among them, embraced a version of this view. Piaget is commonly thought to have endorsed a similar idea. So when *Math Their Way* argues that the key to teaching math for understanding is to get children to use the right sorts of activities and materials, it is in one of the main lines of modern educational thought and practice.

The book's claim also helps to explain why it gives so little attention to the explanation of mathematical ideas. For the author seems convinced that it is superfluous.

Appropriate materials and activities alone will do the trick. Students will "understand" math without any need to question or explain mathematical ideas. This made *Math Their Way* an appealing package, for it enabled Mrs. O to whole-heartedly embrace teaching math for understanding, without considering or reconsidering mathematics. She was keen that children should understand math and worked hard at helping them. But she placed nearly the entire weight of this effort on concrete materials and activities. Her use of the materials, insisting that all the children actually feel them and perform the same prescribed physical operations with them, suggests that she endowed them with enormous, even magical, instructional powers. The lack of any other ways of making sense of mathematics in her lessons was no oversight. With *Math Their Way*, she simply saw no need for anything else.

In what sense was Mrs. O teaching for understanding? The question opens up a great puzzle. Her classes exuded traditional conceptions of mathematical knowledge and were organized as though explanation and discussion were irrelevant to mathematics. But she had changed her math teaching quite dramatically. She now used a new curriculum specifically designed to promote students' understanding of mathematics, as opposed to simple memorization. And her students worked with materials that represented mathematical relationships in the concrete ways that the Framework and many other authorities endorse.

New Topics and Old Knowledge

The puzzle was apparent in other features of Mrs. O's teaching. For instance, she taught several topics that would not have been covered in many traditional math classes, among them estimation. She told me that estimation is important because it helps students to make sense of numbers by making educated guesses and figuring out why some guesses are better than others. She reported that she dealt with estimation recurrently in her second-grade classwork, for it could not be learned by doing it once or twice, and is useful in many different problem-solving situations. Her reasoning on this matter seemed to accord with the Framework's call for "guessing and checking the result" as an important element in mathematical problem solving (CSDE, 1985, p. 14).

But the teaching that I observed did not entirely realize these ambitions. In one lesson, Mrs. O asked the class to estimate how many large paper clips would be required to span one edge of her desk. Two students were enlisted to stand near the desk and hold up the clips. They were near enough to visually gauge its width in relation to the clips, but all the other students remained at their tables, scattered around the room. None had any clips, and few could see the edge of the teacher's desk that was in question, for it was a side edge, away from most of the class. Seated at the back with many of the kids, I could see that they were the large sort of clip, but even then they were barely visible.

So only two members of the class had real contact with the two key data sources in the problem—visible, palpable clips and a clear view of the desk edge. Hence only two members of the class had any solid basis for deciding if their estimates were mathematically reasonable. Even Mrs. O was seated too far away to see the edge well.

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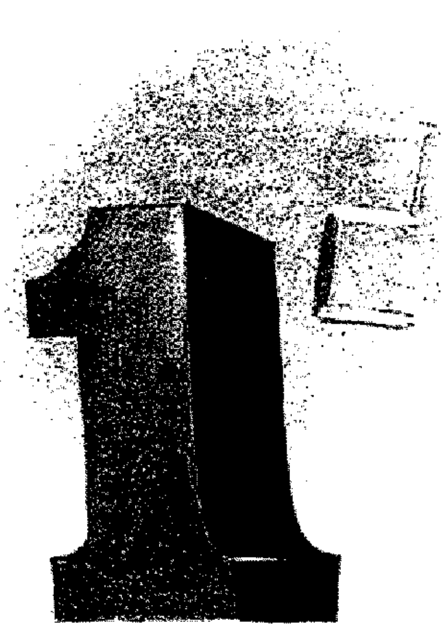

The problem was sensible and could have been an opportunity to make and discuss estimates of a real puzzle. But it was set up in a way that frustrated mathematical sense making.

Mrs. O did not seem aware of this. She asked the students to estimate how many clips it would take to cover the edge and to write down their answers. Then she took estimates from most of the class, wrote them on the board, and asked class members if the estimates were "reasonable." Not surprisingly, many of the answers lacked mathematical discrimination. Estimates that were close to three times the actual answer, or one-third of it, were accepted by the class and the teacher as "reasonable." Indeed, no answers were rejected as unreasonable, even though quite a few were far from the mark. Nor were some estimates distinguished as more or less reasonable than others. Mrs. O did ask the class what "reasonable" meant, and one boy offered an appropriate answer, suggesting that the class had some previous contact with this idea.

I could see nothing that led inexorably to this treatment. Mrs. O had many clips. If eight or ten had been passed around, the kids would have had at least a bit of direct access to one element in the estimation problem. She also could have pointed to the desk edge that the class could see, rather than the far edge that was obscured from their view. Alternatively, she could have invited them to estimate the length of their own desk edges, which were all the same standard-issue models. Either or both would have given them much more direct contact with the elements of the problem and more of a basis to consider how reasonable their estimates were.

Why did Mrs. O not set the problem up in one of these ways? In an interview after the class, she displayed no sense that anything had been wrong, in response to my queries. She seemed to understand the broad purpose of teaching and learning estimation, but she taught as though she lacked the mathematical and pedagogical infrastructure—the knowledge of mathematics, and of teaching and learning mathematics—that would have helped her to set the problem up so that the crucial mathematical data were available to students. And despite her earlier comments, Mrs. O presented estimation as a topic in its own right rather than as a part of solving problems that came up in the course of studying mathematics. It was as though she thought that estimation bore no intimate relation to solving the ordinary run of mathematical problems. In contrast, the Framework argued that ". . . estimation activities should be presented not as separate lessons but as a step to be used in all computational activities" (CSDE, 1985, p.4).

I wondered what students made of this. They appeared to accept the lesson as reasonable. No one complained about the lack of comprehensible data, which they might have done if they were used to such data. No one said that they had done it differently some other time and that this didn't make sense. That could mean that the other lessons on estimation conveyed a similar impression, or it could mean that students were doing as they had been told because they had so often been told to do so, or because they had a visitor. Or it may mean only that students took nothing from the lesson. Schools present many mystifying examples of adult behavior that children learn to simply accept, and this may have been such a case.

Was this teaching math for understanding? From one angle, it was. Mrs. O taught a novel and important topic, specifically intended to promote students' sense-making in arithmetic. It may have done that. But from another angle it was not. For the problem was framed so that many students could not bring mathematical evidence to bear on it and had little basis for making reasonable estimates. These alternatives are not mutually exclusive. This bit of teaching could have promoted more understanding of mathematics along with more misunderstanding.

New Organization and Old Discourse

Mrs. O's class was organized to promote "cooperative learning." The students' desks and tables were gathered in groups of four and five so that they could easily work together. Each group had a leader to help with various chores, and instructional materials often were managed by groups rather than individually. The new Framework approved: "To internalize concepts and apply them to new situations, students must interact with materials, express their thoughts, and discuss alternative approaches and explanations. Often, these activities can be accomplished well in groups of four students" (CSDE, 1985, p.16).

Hence cooperative learning groups are seen as vehicles for a new sort of instructional discourse, in which students would do much more of the teaching. Students would learn from their own efforts to articulate and explain ideas, and they would learn from their mates' ideas. The Framework explains: "Students have more chances to speak in a small group than in a class discussion; and in that setting, some students are more comfortable speculating, questioning, and explaining concepts in order to clarify their thinking" (CSDE, 1985, pp. 16-17). Mrs. O's class was spatially and socially organized for such learning, but the class was conducted in a highly structured and classically teacher-centered fashion. The exchanges were either between the teacher and one student or choral responses to the teacher's questions. No student ever spoke to another about mathematical ideas, as part of the public discourse of the classes that I observed. Nor was such conversation ever encouraged by the teacher. Indeed, Mrs. O specifically discouraged students from speaking with each other in her efforts to keep the class orderly and quiet.

Still, the small groups were used for some instructional purposes. In one class that I observed, Mrs. O announced a "graphing activity" about mid-way through the math period. She wrote across the chalk board, at the front of the room, "Letter to Santa?" Underneath she wrote "Yes" and "No." Then she told the children that she would call on them by groups to answer the question. If she had been following the Framework's injunctions, she might have asked each group to tally its answers to the question, asked each group to figure out whether it had more "yes" than "no" answers, or the reverse, and asked each group to figure out how many more. Then she might have had each group contribute its totals to the chart at the front of the room. This would not have been the most challenging group activity, but it would have meaningfully used the small groups as agents for working on this

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bit of mathematics.

But Mrs. O used the groups only to call on individual children. She asked individuals from each group to come to the front and put their entry under the "Yes" or "No" column, exhausting one group before going on to the next. The groups were used in a socially meaningful way, but there was no mathematical discourse within them.

Was this teaching for understanding? Mrs. O did use a new form of classroom organization that was designed to promote collaborative work and broader discourse about academic work. She did employ the small groups consistently during my visits. The children seemed quite familiar with procedures and worked easily in this organization. She also used the groups to distribute and collect instructional materials and to dismiss the class for lunch and recess (she let the quietest and tidiest group go first). Moreover, she referred to her classwork as "cooperative learning" and used the organization for some regular features of classroom work. When I mentally compared her class with others I had observed in which students sat in rows and in which there was only whole group or individual work, her class seemed really different. But she filled the new social organization with old discourse processes that effectively frustrated the sort of cooperative learning that the Framework's authors had envisioned. I asked if she ever used the groups for discussions and that sort of thing; she said that mostly she worked in the ways I had observed.

REPRISE

I have emphasized certain tensions within Mrs. O's classes, but these came into view partly because I crouched in her class with one eye on the Framework. Other observers might not have noticed them, for Mrs. O's lessons went quite smoothly. She and her students were well used to each other, and the contrary elements of instruction that I have highlighted did not jar the class. On the contrary, students and teacher acted as though these lessons made perfect sense. Features of instruction that seemed at odds analytically appeared to co-exist nicely in practice.

One reason for this lay in the classroom discourse. Mrs. O never invited or permitted broad participation in mathematical discussion. She held most exchanges within a recitation format; she initiated nearly every interaction, and the students responded. They complied. After all, most second graders want to please their teacher, and compliance is easier than initiation. In consequence, the discourse was very familiar to members of the class, almost ritually so. The calendar exercises that I observed were so familiar that students often gave the answers before she asked the questions. Most of the class participated, but they did so on a narrow track in which she maintained control of direction, content, and pace.

In contrast, the Framework argued that children need to express and discuss their ideas in order to understand the material on which they are working (CSDE, 1985, pp. 14, 16). But the discourse in Mrs. O's class discouraged students from reflecting on mathematical ideas or from sharing their puzzles with the class. Attention was focused instead on successfully managing a highly structured set of activities. This restricted even the questions and ideas that could occur to students, for thought is created, not merely expressed, in social interactions. Mrs. O employed a curriculum that sought to teach math for understanding, but she kept evidence about what students understood from entering the classroom discourse. The discourse remained smooth partly because so much possible roughness was choked off at the source.

Another reason for the lesson's smoothness lay in Mrs. O's knowledge of mathematics. Though she plainly wanted her students to understand this subject, she did not know mathematics deeply or extensively. She had taken one or two courses in college, and reported that she had liked them; but she had not pursued the subject further. Moreover, Mrs. O knew mathematics as a fixed body of truths, rather than as a particular way of framing and solving problems. Questioning, argument, and explanation seemed quite foreign to her. She worked hard to make the fixed truths accessible to her students, using a new curriculum that promised to embody mathematical ideas and operations in concrete materials and physical activities. This struck her (and many other teachers today) as a great improvement on words and sheets of numbers. But neither Mrs. O nor *Math Their Way* saw mathematics as a source of puzzles, as a terrain for argument, or as a subject in which questioning and explanation were key elements of learning—all ideas that are plainly featured in the Framework (CSDE, 1985, pp.13-14). Lacking a sense of importance of explanation in mathematics, she simply slipped over many opportunities to elicit it, unaware that they existed. Because her conception of mathematical understanding was so limited, she could "teach for understanding," with little sense of how much remained to be understood, how much might be incompletely or naively understood, and how much might still remain to be taught. Working as she did near the surface of the subject, many elements of understanding and many pedagogical possibilities remained invisible. Mathematically she was on thin ice. But she did not know it and so skated smoothly on with great confidence.

Mathematically she was on thin ice.

In a sense, then, the tensions that I observed were not there. Though real enough in my view, they did not enter the public arena of the class. For they were kept hidden by the nature of the class itself. Mrs. O's modest grasp of mathematics, her limited conception of mathematical understanding, and her close management of classroom discourse simply obliterated many potential sources of roughness in the lessons. Had Mrs. O known more math and constructed a somewhat more open discourse, her class would not have run so smoothly. Some of the tensions that I noticed would have become audible and visible. Things would have been rougher, potentially more fruitful, and vastly more difficult.

PRACTICE AND PROGRESS

Is Mrs. O's mathematical revolution a sign of progress or confusion? Does it signal an advance or a setback for the latest new math? It probably is unwise to sharply distinguish progress from confusion when considering such deep change in instruction as reformers press today. For the teachers and students who try to carry out such change cannot simply shed their old ideas and practices like a shabby coat and slip on something new. Inherited ideas and practices are all that teachers and students know, even as they begin to know something else. As they reach out toward a new instruction, they do so with their old instructional practices. Their past is their only path to the future. Mixed practice and confusion, therefore, seem essential to progress.

This point often goes unnoticed by those in the throes of change, as well as by those who promote it. The changes in Mrs. O's teaching that seemed paradoxical to me seemed revolutionary to her, and I do not think she was deluded. She saw certain crucial limits of her early emphasis on computation and memorization and was convinced that her classes have greatly improved. She contended that her students now understood and learned much more math than their predecessors had a few years ago. She even asserts that this has been reflected in their achievement test scores. I have no direct evidence of these claims. But when I mentally compared this class with others that I have seen, in which instruction consisted only of rote exercises in manipulating numbers, her claims seemed entirely plausible. Many traditional teachers viewing her classes today would also think they were revolutionary.

But all revolutions preserve large elements of the old order as they invent new ones, if only because everything cannot change at once. One continuing element in Mrs. O's practice was a conception of mathematics as a fixed body of knowledge. Another was a view of learning mathematics as getting the right answers. She said that math had not been a favorite subject in school and that she had only learned to do well at it in college. When I asked her how that had happened, she said, ". . . I found that if I just didn't ask so many 'why's' about things that it all started fitting into place. . . ." Mrs. O learned to do well at math by avoiding exactly the sort of questions that the Framework associates with understanding mathematics. She noted that her view of math has not changed since college.

Another persistent element in her practice was "clinical teaching," that is, the California version of Madeline Hunter's Instructional Theory Into Practice (ITIP). This approach stresses the importance of structure in lessons and is associated with a rigid, sonata-form pedagogy, close teacher control, brisk pacing, and highly structured recitations. ITIP appears to have played an important part in Mrs. O's own education as a teacher, and she has been encouraged to persist with it. Both her principal and assistant principal at the time were devotees of Hunter's method and vigorously promoted it in the

Is Mrs. O's mathematical revolution a sign of progress or confusion?

school. I asked all three of them whether clinical teaching worked well with the Framework. None saw any inconsistency, saying emphatically that the two innovations were "complementary." Yet as ITIP was realized in Mrs. O's class, it cut across the grain of the Framework. For she took clinical teaching as a license to rigidly limit discourse, to closely control social interaction, to focus the classroom on herself, and to hold instruction to relatively simple objectives.

If Mrs. O's past affected the changes in her practice, it also affected how she saw them. In the spring of 1989, I asked where her math teaching stood. She thought that her revolution was over. Her teaching had changed definitively: She had arrived at the other shore. In response to further queries, Mrs. O evinced no sense that there were areas in her math teaching that needed improvement. Nor did she seem to want guidance about how well she was doing or how far she had come.

There is an arresting contrast here. From an observer's perspective, especially one who had the new Framework in mind, Mrs. O looked like a teacher in transition. On this view, she might be imagined near the beginning of growth toward new math teaching. But the matter looked quite different to Mrs. O, who considered things in light of her past work. She saw herself as a teacher who had made a great transition and mastered a new practice.

Which perspective is most appropriate—Mrs. O's or the hypothetical observer's? This is a terrific puzzle. One wants to honor this teacher, who has made a serious and sincere effort to change, and who has changed. But one also wants to honor efforts to achieve greater intelligence and humanity in mathematics instruction.

We might begin by noticing that Mrs. O had only one perspective available. No one had asked how she saw her math teaching, in light of the Framework, nor had she been offered opportunities to view other sorts of teaching. If no one in California education had seen fit to ask her the question and help her to figure out answers, could we expect her to have asked and answered it all alone?

If math teaching is half as deficient as reformers say, then few teachers would know enough to raise many fruitful questions about their practice. Mrs. O's own lessons quite effectively protected her from experiences that might have provoked such questions. But even if such questions were somehow raised for Mrs. O and other teachers, would they know enough to frame appropriate answers? How could teachers be expected to assess their own progress in inventing a new sort of instruction if their teaching is half as dismal as reformers suggest?

One can imagine arrangements that would help teachers to learn more about math teaching and how to think about it. But California's budget for professional development is painfully modest just now. Lacking such assistance, could teachers assess their progress as though they had access to thoughtful commentary, when in fact most had none?

Even if Mrs. O had had such assistance, she would still have had to build on her past practices as she changed, like any practitioner. Hence her view of how much she had accomplished would be tied to her subjective experience of change. Teachers whose practice is very traditional would most likely think that their first steps—that would seem small to an observer—were quite large. For from a perspective still rooted mostly in a traditional practice, such modest changes would be immense. Such teachers might come to regard them as small only if they

From this perspective, Mrs. O's progress seems remarkable.

took some larger steps later on and consequently gained a different perspective. Of course, we might expect more from teachers who had a good deal of help in thinking about teaching in some active discourse about their work, in which questions were asked and answered from a variety of perspectives. For those teachers would have more resources for change, unlike colleagues who had been left to figure things out for themselves.

What would it take to make such assistance available to teachers? And to help teachers pay constructive attention to it? Neither query has been given much attention, either in efforts to change instruction or in efforts to understand such change. But without such help, it is difficult to imagine how Mrs. O and many other teachers could make the changes that reformers now invite.

POLICY AND PRACTICE

Mrs. O's math classes suggest a paradox. On the one hand, policy is the key to changing practice. For new instructional policies illuminate deficiencies in teaching and learning and provide impetus for change. From this perspective, teachers are the problem, for it is their mechanical and modest knowledge of mathematics that impedes progress. But teachers also are the chief agents of any new instruction, because few students will learn a new mathematics unless teachers teach it. The new policy seeks great changes in knowledge, learning, and teaching, yet these are intimately held human constructions. They cannot be changed unless the people who know, teach, and learn want to change, take an active part in changing, and have the resources to change.

How can practice be improved if the chief change agents are also are the problem to be corrected? This puzzle is worth noticing partly because so much instructional policy making seems to ignore it. Many policies that seek fundamental instructional reform look as though their authors believed that students and teachers would change if they simply were told to do so. New goals are articulated, and exhortations to pursue them are issued. Sometimes new materials are provided. Another reason to notice the paradox is that the instructional changes reformers seek are immense. If the recent reforms are to succeed, students and teachers must not simply absorb a new "body" of knowledge. Rather, they must acquire a new way of thinking about knowledge and a new practice of acquiring it. They must cultivate strategies of problem solving that seem to be quite unusual among adult Americans. They must learn to treat knowledge as something they construct, test, and explore, rather than as something they absorb and accumulate. Additionally, and in order to do all of the above, they must un-learn much of what they know, whether they are second graders or veteran teachers. Their extant knowledge may be naive, but it often works. A few can learn these things easily, and some even seem to pick it up on their own. But many very able learners have great difficulty, and so prefer the traditional sorts of learning that reformers reject.

Learning a new mathematics is much more formidable for teachers than students, for they must learn how to teach anew while relearning what to teach. And they must un-learn the mathematics and teaching practices that they have used for decades.

Mrs. O was not taught about the new Framework in a way that recognized these difficulties. Instead, the California state education department taught her about the new math using roughly the same traditional pedagogy that it criticized in the Framework. Like students in many traditional math classrooms, she was told to do something. She was told that it was important. And a synopsis of what she was to learn was provided in a text. The state advanced an instructional revolution, but it used an old pedagogy to do so. If, as the Framework argues, it is implausible to expect students to understand math simply from telling it to them, why is it any less implausible to expect changes in teaching to result simply from telling teachers to change? From this perspective, Mrs. O's progress seems remarkable.

What more might it take to support major instructional change? It is no answer to the question, but I note that few people in Mrs. O's vicinity seemed to be asking that question, let alone taking action based on some answers.

This is no argument against the changes that reformers press. The revised California Mathematics Framework offers a bold and ambitious vision of mathematics instruction, one that took imagination to devise and courage to pursue. Yet this admirable initiative has done little to augment teachers' capacities to realize the vision. The new Framework, for instance, had barely been announced in her school. She knew that it existed but was not sure that she ever had read it. She knew that the principal had a copy and that the new text series had been written in light of the Framework. She had attended a publisher's workshop on the new text and found it informative. She also had studied the text and the teacher's guide. But like many teachers in her district, she used the new book only a little, preferring *Math Their Way*. The state education department also supported a network of teacher development projects, mostly in universities, that offered math workshops for teachers. But there are only a handful of these projects compared with the tens of thousands of teachers in California, and most workshop sessions are brief. A few project staffers follow teachers back into school and offer support for change, but most do not. To the extent that there was support or guidance for change in Mrs. O's practice, it was local, but there was precious little of that.

Hence the changes in Mrs. O's practice were at best weakly guided and supported by the new policy. From one angle, this seems admirable. Mrs. O has had considerable discretion to change her teaching, and she has done so in ways that seem well adapted to her school. Though I call attention to the mixed quality of her teaching, her superiors celebrate her work. But if we take seriously reformers' arguments about the importance of mathematics and the need for a new mathematical pedagogy, then Mrs. O's situation is troublesome. When I observed what I report here, there seemed little chance that she would be helped to struggle through to a more complex knowledge of mathematics and a more complex practice of teaching mathematics. And if she cannot struggle through, how can she better help her students to do so? The recent reform movement has vastly expanded Mrs. O's obligations in teaching mathematics, without much increasing her resources for meeting those obligations. Ambitions for reform have continued to escalate as state and local budgets contract.

That collision between ambitions and resources may turn out to be crippling. Researchers and other commentators on education have begun to appreciate how difficult it is for many students to achieve deep understanding of a subject. This appreciation is at least occasionally evident in the rhetoric of reform. But so far, there is little appreciation of how difficult and costly it will be for teachers to learn new practices in which students are competently guided toward deeper understanding. □

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