PROPORTIONALITY & SIMILAR FIGURES WORKSHOP 1: DISCOVERY

Agenda for Two-Hour Workshop

20 minutes ***Workshop Facilitator/Site Leader***
Introduction

Introduce The Missing Link workshop series.
Hand out the workshop guide and the materials for Workshop 1.
Discuss the following questions:
- What is proportional reasoning?
- How is proportional reasoning used to solve problems?
- Why is this concept important for students to know?

60 minutes ***Whole Group***
View Workshop 1: Proportionality & Similar Figures — Discovery

While watching the program, consider the following focus questions:
- What makes similar figures similar?
- What does similarity have to do with proportional reasoning?
- What are the advantages of the Launch-Explore-Summarize Teaching Model (page 152)?
- What is different about this model from your own teaching methods?
- In the videotaped classroom segments, how can you tell that the students are engaged? What evidence do you have that they are learning?

30 minutes ***Small Groups or with a Partner***
Read, Do and Discuss

Read Lesson 1: Rubber Band Stretchers.
Do Lesson 1.
Discuss findings, focusing on the above questions.

10 minutes ***Workshop Facilitator/Site Leader***
Homework Assignment

- Do Lesson 2: Drawing Similar Figures on a Grid.
- Read the two extension lessons on scale factor. Look at the sample student work related to the second extension lesson (page 32).
- Review the Launch-Explore-Summarize Teaching Model (page 152).
- Review the Why This Topic Matters section (page 37).
- Use your journal to reflect on the focus questions from this workshop. Describe how these lessons deepened your content knowledge about Proportionality & Similar Figures. What did you learn? What else do you need to learn?

BEFORE WATCHING THIS PROGRAM ...

- Make sure to have copies available of Lesson 1: Rubber Band Stretchers (page 11).
- Have materials for Lesson 1:
  - Angle rulers or protractors
  - Rulers
  - Transparencies, transparent centimeter grids and overhead markers
  - Rubber bands
  - Masking tape and blank paper
  - Copies of figures to be enlarged.
- Have materials for Lesson 2 for teachers to take home:
  - Angle rulers or protractors
  - Rulers
  - Grid paper
  - Coordinate points chart.
- Have copies of the extension lessons (page 24) for teachers to take home.

PROPORTIONALITY & SIMILAR FIGURES WORKSHOP 1: DISCOVERY

Agenda for Four-Hour Workshop

20 minutes ***Workshop Facilitator/Site Leader***
Introduction

Introduce The Missing Link workshop series.
Hand out the workshop guide and the materials for Workshop 1.
Discuss the following questions:
- What is proportional reasoning?
- How is proportional reasoning used to solve problems?
- Why is this concept important for students to know?

30 minutes ***Whole Group***
View Lesson 1: Rubber Band Stretchers

While watching the program, consider the following focus questions:
- What makes similar figures similar?
- What does similarity have to do with proportional reasoning?
- What are the advantages of the Launch-Explore-Summarize Teaching Model (page 152)?
- What is different about this model and your own teaching methods?
- In the videotaped classroom segments, how can you tell that the students are engaged? What evidence do you have that they are learning?

20 minutes ***Small Groups or with a Partner***
Read, Do and Discuss

Read Lesson 1: Rubber Band Stretchers.
Do Lesson 1.
Discuss findings, focusing on the above questions.

20 minutes ***Whole Group***
Discuss

Share findings from small groups, focusing on the mathematics of the lesson.

30 minutes ***Whole Group***
View Lesson 2: Drawing Similar Figures on a Grid

While watching the program, consider the following focus questions:
- What is the connection between the algebraic rule and the geometric shapes?
- What tools would students need to make numerical comparisons between shapes?
- How could students compare the areas of the Flip Family figures?
- What is the relationship between scale factor and area?
- What are the key math concepts that students should understand with this lesson?

BEFORE WATCHING THIS PROGRAM ...

- Make sure to have copies available of all the lessons in this workshop.
- Have materials for Lesson 1:
  - Angle rulers or protractors
  - Rulers
  - Transparencies, transparent centimeter grids and overhead markers
  - Rubber bands
  - Masking tape and blank paper
  - Copies of figures to be enlarged.
- Have materials for Lesson 2:
  - Angle rulers or protractors
  - Rulers
  - Grid paper
  - Coordinate points chart.
- Have materials for Extension 1:
  - Rulers
  - Small photographs
  - Large sheet of dot paper
  - Calculators
  - Large sheet of chart paper.

20 minutes ***Small Groups or with a Partner***
Read, Do and Discuss

Read Lesson 2: Drawing Similar Figures on a Grid.
Do Lesson 2.
Discuss findings, focusing on the above questions.

20 minutes ***Whole Group***
Discuss

Share findings from small groups, focusing on the mathematics of the lesson.

60 minutes ***Whole Group***
Read, Do and Discuss

Read Extension 1: Enlarging a Picture and Scaling Up and Extension 2: The Carly Problem. Spend some time discussing scale factor

- What is scale factor?
- How do you find scale factor given two similar figures?

Do Extension 1: Enlarging a Picture and Scaling Up.
Discuss.

20 minutes ***Workshop Facilitator/Site Leader***
Homework Assignment

- Review the two extension lessons on scale factor. Look at the sample student work related to the second extension lesson (page 32).
- Do The Carly Problem on your own.
- Review the Launch-Explore-Summarize Teaching Model (page 152).
- Review the Why This Topic Matters section (page 37).
- Use your journal to reflect on the focus questions from this workshop. Describe how these lessons deepened your content knowledge about Proportionality & Similar Figures. What did you learn? What else do you need to learn?
- How can you make this topic relevant for your students?
- What questions do you have about the lessons?

PROPORTIONALITY & SIMILAR FIGURES WORKSHOP 1: DISCOVERY

■ Lesson 1: Rubber Band Stretchers

A. The Big Ideas

Purpose of This Lesson

This lesson introduces students to the concept of proportional reasoning by having them make numerical comparisons. It also helps students understand what makes similar figures similar.

Mathematical and Problem-Solving Goals

- Explore similar figures.
- Recognize similar figures and begin to identify the characteristics of similar figures.

Connections to NCTM Standards

- Use proportions to examine relationships between similar plane figures.
- Develop, analyze and explain methods for solving problems involving proportions (e.g., scaling, finding equivalent ratios).
- Develop a disposition to formulate, represent, abstract and generalize in situations within and outside mathematics.
- Build new mathematical knowledge through work with problems.
- Make and investigate mathematical conjectures.

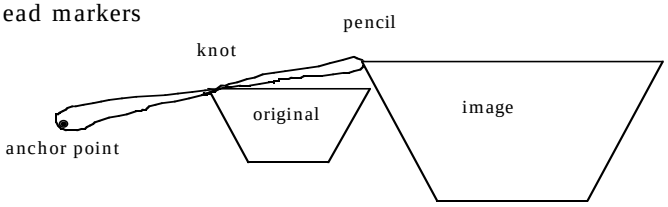
B. The Lesson

Recommended Mathematical Background

- Reading a ruler
- Measuring angles (with protractors and/or angle rulers)

Materials

- Angle rulers or protractors
- Rulers
- Transparencies, transparent centimeter grids and overhead markers
- Rubber bands
- Masking tape and blank paper
- Copies of figures to be enlarged



Time

- 80 minutes

Lesson Overview

In this investigation, students produce images of original drawings using two identical rubber bands tied together to form a rubber band stretcher. By measuring sides, areas and angles and by comparing the image to the original figure, they begin to explore the concept of proportionality.

Students work independently or in small groups on this lesson, using the handouts on the following pages. They put one end of the stretcher on the anchor point and hold it down securely, without covering up any

TEACHING TOOLS

- As you discuss and use this lesson, make sure to take advantage of the Teacher Planning Tools (pages 151-157):
  - Planning a Math Unit: Launch-Explore-Summarize Teaching Model
  - Lesson Planner Template
  - Questions to Stimulate Student Thinking
  - Guidelines for Grouping.
- Refer to Why This Topic Matters (page 37) to help make this topic relevant to your students.
- Check out our Web site (www.learner.org/channel/workshops/missinglink) for additional tools and resources — and to join The Missing Link online discussion forum.

more of the band than necessary. They then put the pencil through the other end and stretch the band until the knot is just above part of the original figure. Move the pencil as the knot traces the figure. It is important to keep the knot directly over the original figure as the pencil draws the new figure. Have a variety of tools and supplies available for students to explore the figures. Encourage students to measure and record the side lengths, angle measures, and areas of both the original figure and its image. Then compare how line lengths, areas and angles change. Students also will make and test predictions about how a drawing would be affected if they were to move the anchor point up, down or farther away from the original figure.

Launch*

Before beginning this lesson, your students might need some time to practice tying the rubber bands and manipulating the apparatus. Have them stand and sight directly down over the stretcher and keep focused on the knot following the figure — not the new drawing. Have them experiment using a variety of writing utensils — pencil, crayon, marker — to find what works best for them. Reassure them that, at this point, it's not necessary to stress precision. The goal in this first lesson is to get students exploring the concepts of ratio and similar figures. Your students also might need a mini-lesson in how to use angle rulers or protractors.

Explore*

This is the part of the workshop where students "get messy with the math." Make sure students have lots of "tools" to make comparisons (rulers, protractors, blank and gridded transparencies). One of your primary jobs is to make sure they don't miss the math concepts behind the activity — you don't want them to come away thinking it was a game or an art project. Make sure to have lots of questions to use with students as you circulate among the groups: What do you see? Why might these two drawings be different? Do you think using a crayon instead of a pencil will have much impact on the results? What measurements can be made to compare the two figures? How is the large figure related to the small figure? Is anything the same? How can you compare the areas of the two figures? How many times larger is the image than the original? How do the perimeters compare?

Remember that at this point in the lesson you are there to observe, not to teach. Students who are stuck and students who get it quickly might need extra attention — try to guide them by asking leading questions like those above, rather than "taking over" their project (see Questions to Stimulate Student Thinking, page 155).

Summarize*

Have students report their findings. Make sure they understand the core concepts of this lesson:

- similar figures are enlargements or reductions with the same basic shape
- similar figures have congruent angles and sides that grow or shrink by a constant factor.

When students are reporting their findings, push them to explain more clearly what they saw. "Why do you say that? How did you come to your conclusion?" Getting the right answer is only part of the students' assignment; they also need to be able to communicate the answer clearly. Ask the whole group questions that came up during the Explore phase to generate class discussion.

Even if you extend this lesson over two class periods, make sure to come to some closure at the end of each session. And note how the Summarize of the first lesson leads directly to the Launch of the second lesson.

Part of the purpose of the Summarize is to allow you to assess how well your students are progressing toward the goals of the lesson. Use the discussion to help you determine whether additional teaching and/or additional exploration by students is needed before they go on to the extension lessons.

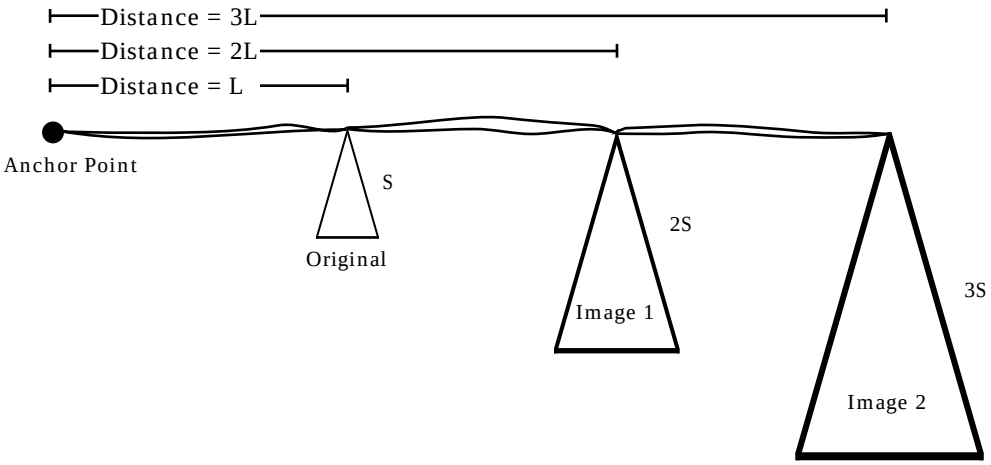
**Throughout the series, Jan Robinson uses the Launch-Explore-Summarize Teaching Model of instruction. It is discussed at some length during this first workshop and in the Teacher Planning Tools on page 152.*

The Mathematics in Constructing Similar Figures with Rubber Band Stretchers

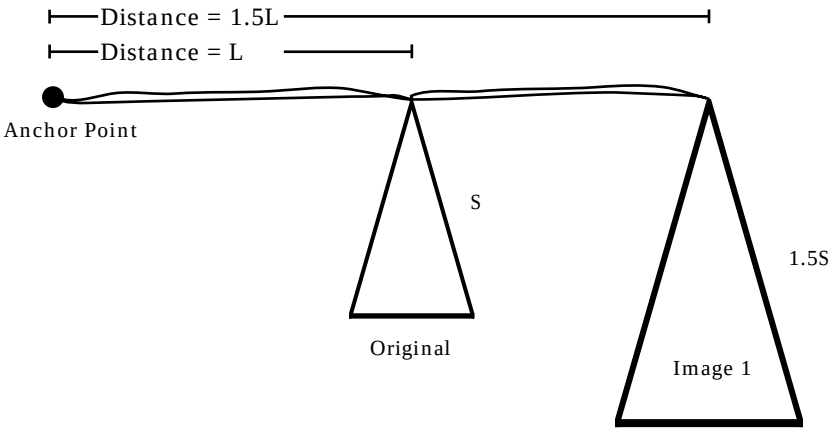
The rubber bands are tools that enable students to construct similar figures easily. There is no magic in the rubber bands themselves. The scale factor (the number of times larger the side of the image is compared to the original) is determined by the following ratio:

$$\frac{\text{distance from the anchor point to the image}}{\text{distance from anchor point to the original}}$$

Therefore, if two identical rubber bands are used, the distance from the anchor point to the image probably will be twice the distance of the anchor point to the original. The length of the sides of the image will be close to two times longer than the sides of the original. If three identical rubber bands are linked together and figures are drawn from the same anchor point, the scale factor (or ratio of the side lengths) will be approximately 3:1.

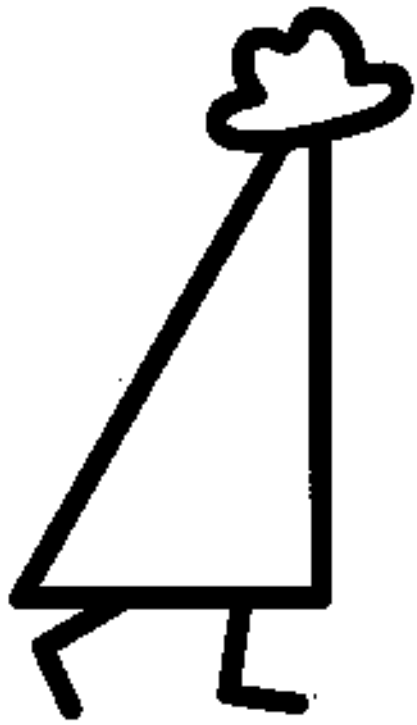


What happens, then, if the rubber bands are different sizes or different thicknesses? The key is the effect the different rubber bands have on how far away the image is drawn from the anchor point. If the second rubber band is the same thickness as the first, but only half the size, the sides of the image probably will be only 1.5 times longer than the original figure. Experiment with this concept to make sure you understand how the size of the image depends on how far away from the anchor point the image is being drawn.



RIGHT-HANDED VERSION

P is the anchor point



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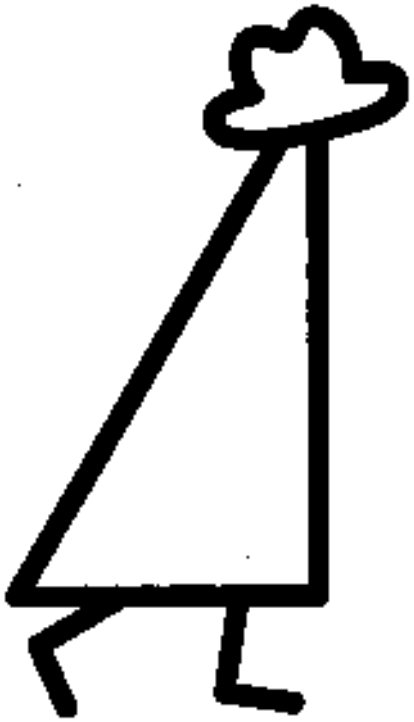
Source: *Connected Mathematics Project – Stretching and Shrinking*

LEFT-HANDED VERSION

P is the anchor point



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Source: *Connected Mathematics Project – Stretching and Shrinking*

RIGHT-HANDED VERSION

P is the anchor point

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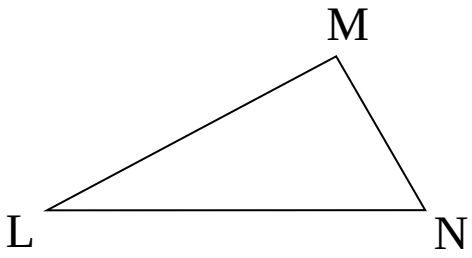

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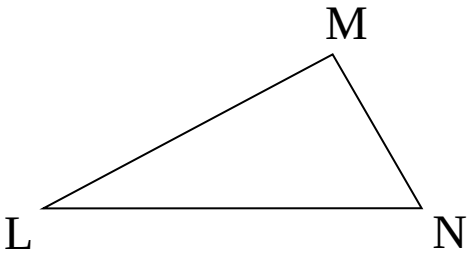
LEFT-HANDED VERSION

P is the anchor point



Source: *Connected Mathematics Project – Stretching and Shrinking*

■ Lesson 2: Drawing Similar Figures on a Grid

A. The Big Ideas

Purpose of This Lesson

Students use ordered pairs and algebraic rules to create similar figures in a coordinate plane. They also make the connection between the algebraic rule used to enlarge the figure and the scale factor of enlargement.

Mathematical and Problem-Solving Goals

- Draw similar figures on a coordinate grid, using algebraic rules.
- Learn that corresponding angles of similar figures are equal.
- Learn that corresponding sides of similar figures are proportional.
- Understand that sides and perimeters of similar figures grow by a scale factor and that the areas grow by the square of the scale factor.

Connections to NCTM Standards

- Learn to use coordinate geometry to represent and examine the properties of geometric figures.
- Use proportions to examine relationships between similar plane figures.
- Understand the concepts of congruence and similarity using transformations.
- Estimate and measure angles, perimeter, area, surface area and volume.
- Create and critique inductive and deductive arguments concerning geometric ideas and relationships.
- Build new mathematical knowledge through work with problems.
- Make and investigate mathematical conjectures.
- Describe size, position and orientation of figures under transformations such as flips, turns, slides and magnification.
- Use geometric models to represent and explain numerical and algebraic relationships.

B. The Lesson

Recommended Mathematical Background

- Graphing ordered pairs in the first quadrant

Materials

- Angle rulers or protractors
- Rulers
- Grid paper
- Coordinate points chart

Time

- 60 minutes

Lesson Overview

In this investigation, students create similar and nonsimilar shapes using a coordinate system. This method is a more precise way of enlarging a figure than was the rubber band stretcher lesson.

Launch

During the Launch, you might need to give your students a very brief refresher lesson on how to plot points on a coordinate grid. Have them share any tips for how they remember which is the x-axis and which is the y-axis.

TEACHING TOOLS

- As you discuss and use this lesson, make sure to take advantage of the Teacher Planning Tools (pages 151-157):
  - Planning a Math Unit: Launch-Explore-Summarize Teaching Model
  - Lesson Planner Template
  - Questions to Stimulate Student Thinking
  - Guidelines for Grouping.
- Refer to Why This Topic Matters (page 37) to help make this topic relevant to your students.
- Check out our Web site (www.learner.org/channel/workshops/missinglink) for additional tools and resources — and to join The Missing Link online discussion forum.

Before they start working on their own, show the whole class how to fill out The Flip Family table and encourage students to predict which figures will be in the family and which will be impostors before they do their drawings. Students will use the idea of "same shape" to discover that similar figures have corresponding angles that are equal and corresponding sides that grow by the same factor (scale factor). After graphing simple cartoon figures, they must decide which figures are members of the same family and which are impostors. Some of the figures are perfect enlargements of the original, while others are distorted by stretching either horizontally or vertically. Students investigate side lengths, angles, areas and perimeters to determine the characteristics of similar figures. Have each student work individually to fill out the table and plot the points.

Explore

Remember Jan Robinson's advice to have three lessons ready: the one you thought you were going to teach, the one for students who are stuck and the one for students who get it quickly. As you circulate among the tables, listen and watch the students at work, and use their ideas to guide your Summarize after the lesson. Ask questions that emphasize the math: How does scale factor affect area? What will happen if we multiply by a scale factor less than 1? How do the angles tell you who are Flips and who are not? How many Zips will fit into Zap? Into Bip? What effect does the algebraic rule have on the figure?

Summarize

Have students report their findings. Make sure they understand the core concepts of this lesson:

- scale factor is the ratio of the side length of the image to the corresponding side length of the original
- if the scale factor is less than 1, the image shrinks; if it is greater than 1, the image is enlarged
- the areas of similar figures grow or shrink by the square of the scale factor.

You might need two class periods to complete this lesson — one for students to fill in and discuss their tables, one to make and explain their drawings. Do a short Summarize at the end of the first period and a recap of the Lesson Overview at the start of the second.

Challenge students to define the words they've been using throughout the lesson: similar, scale factor, congruent, proportion. Which terms are the most mathematically accurate?

Part of the purpose of the Summarize is to allow you to assess how well your students are progressing toward the goals of the lesson. Use the discussion to help you determine whether additional teaching and/or additional exploration by students is needed before they go on to the extension lessons.

Additional Notes

- This lesson presents scale factor in a very simple, concrete format. Your students will be much more likely to remember and be able to apply the concept of scale factor if they've first had a chance to see it in action in this lesson. This is a good lesson to reinforce your students' number sense; for instance, multiplication doesn't always produce a larger number (1/2 x 1/2 = 1/4). Make sure to discuss how these concepts can be used to solve other problems. (See Why This Topic Matters, page 37.)

THE FLIP FAMILY

Plot each set of points separately. Connect the points in each set in order. Use the rules to complete the table. Then plot each new character on a separate grid.

Zip Flip	Zap	Pip	Pop	Bip	Bop
(x, y)	(2x, 2y)	(0.5x, 0.5y)	(3x, y)	(3x, 3y)	(x, 3y)
(4, 2)	(8, 4)				
(4, 10)	(8, 20)				
(6, 14)	(12, 28)				
(7, 12)					
(8, 14)					
(9, 12)					
(10, 14)					
(10, 2)					
(4, 2)					
Start Over					
(4, 4)					
(6, 4)					
(8, 6)					
Start Over					
(6, 8)					
(6, 9)					
(8, 9)					
(8, 8)					
(6, 8)					
Start Over					
(4, 8)					
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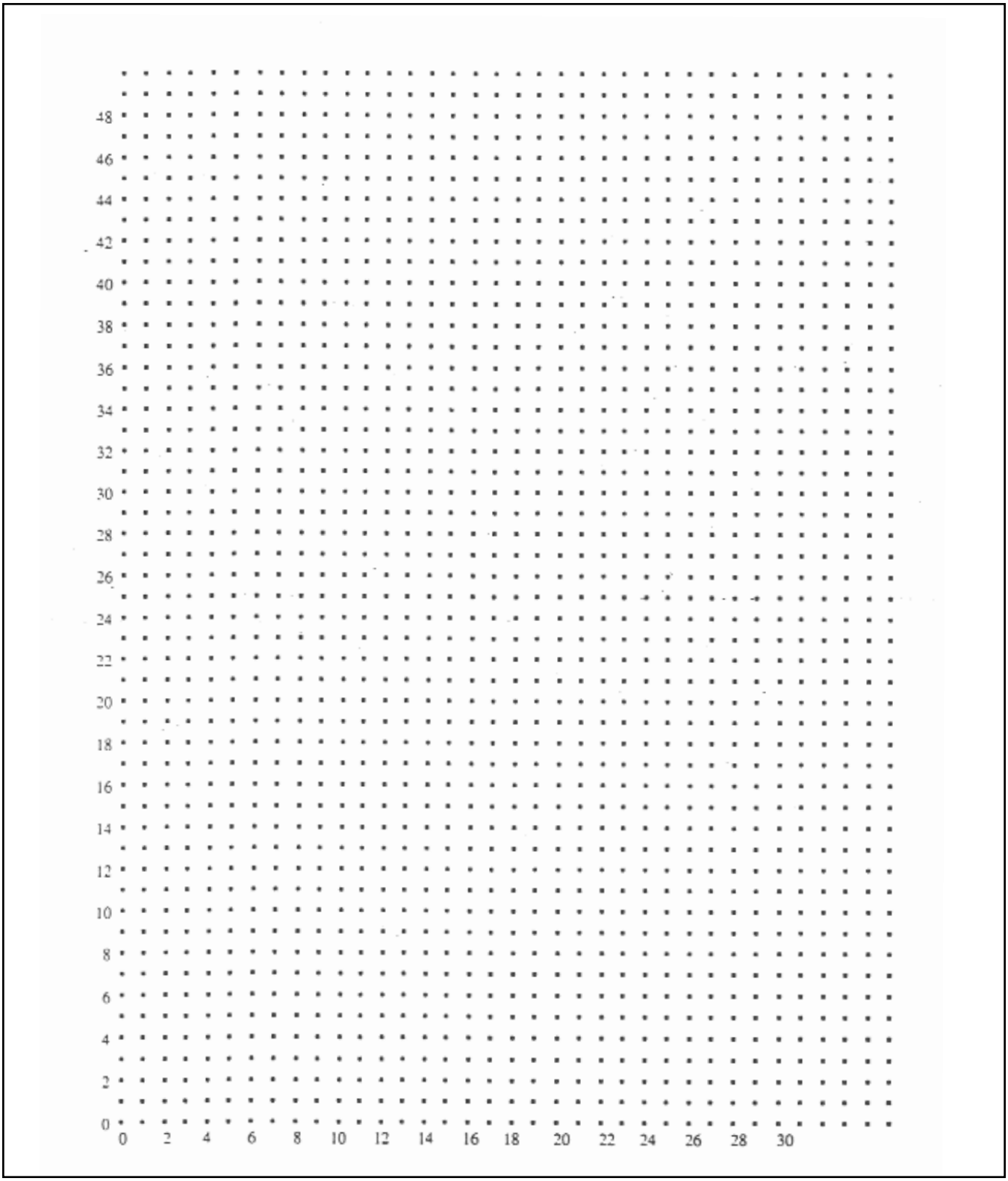
(4, 6)					

Source: Adapted from *Connected Mathematics Project – Stretching and Shrinking*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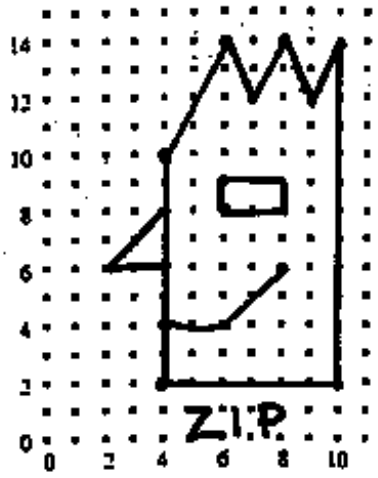
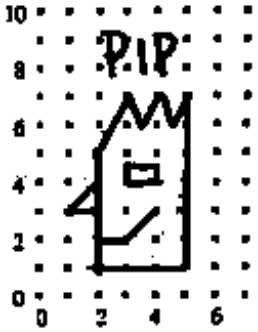
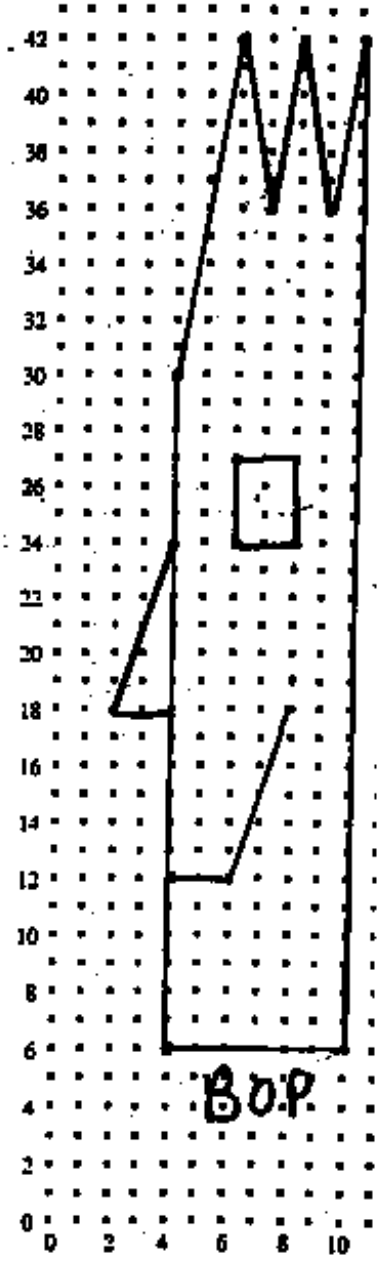
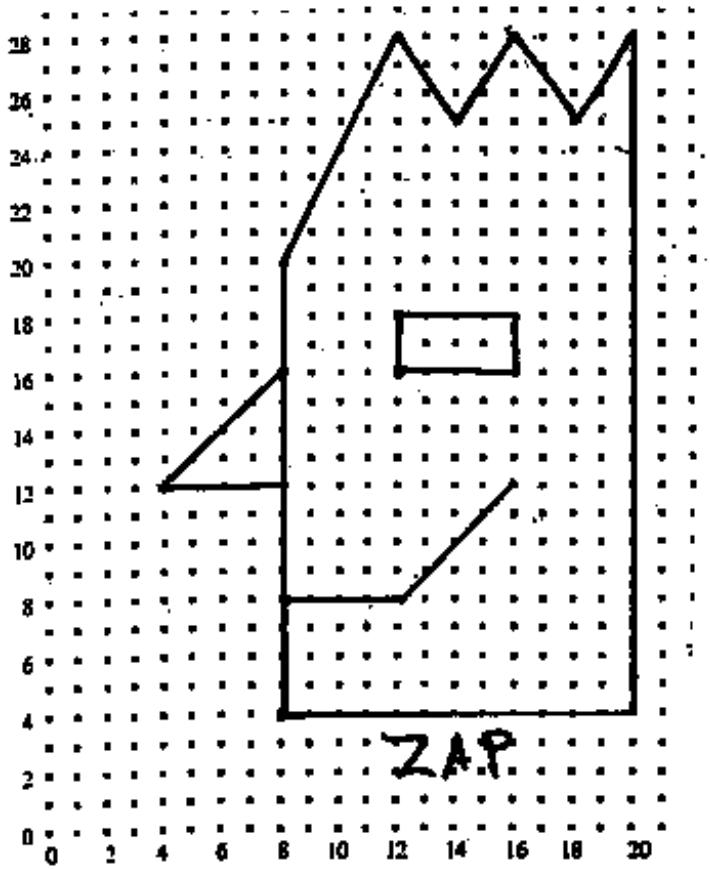
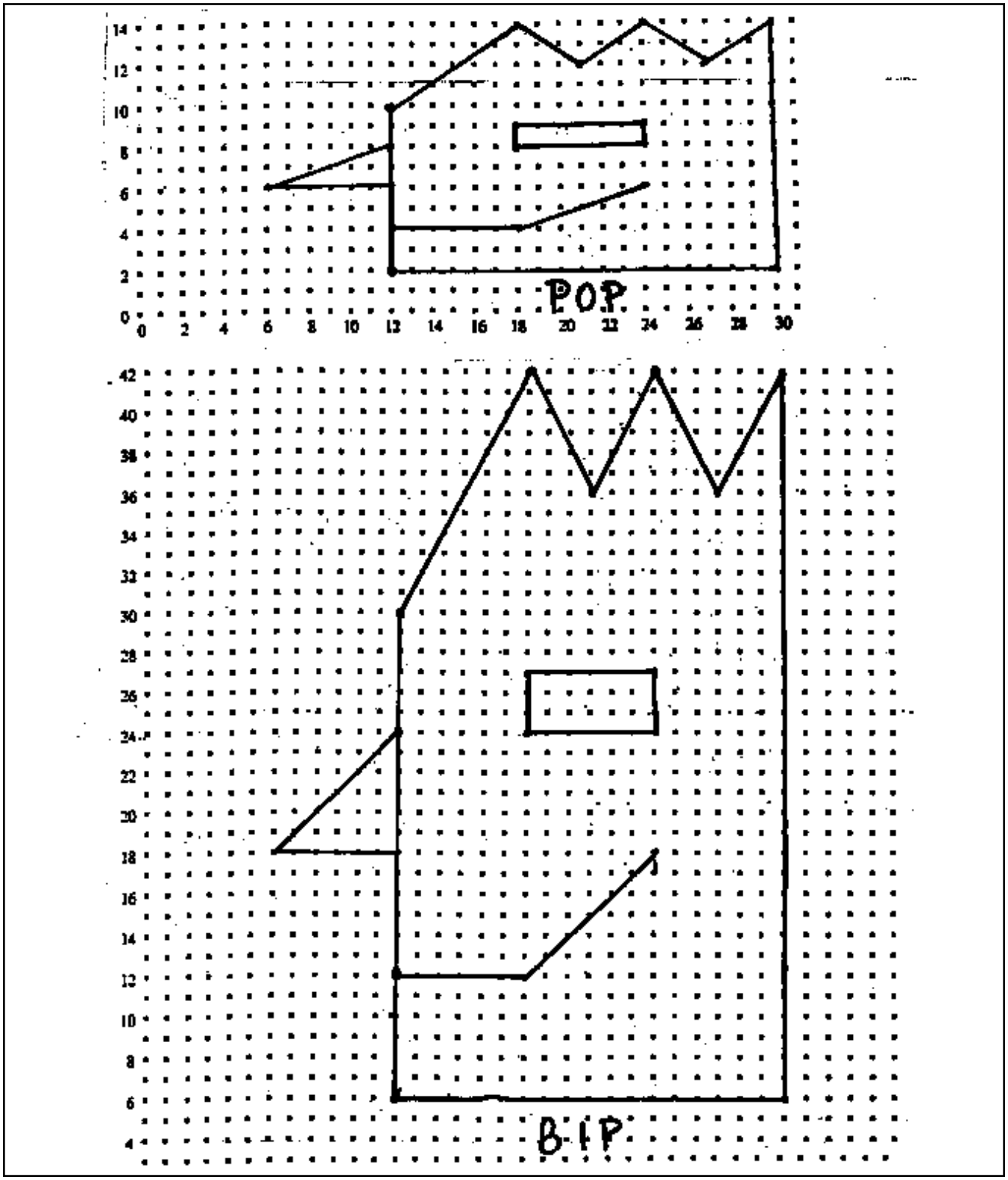
WHICH ARE FLIPS?










POP
BIP

■ Extension 1: Enlarging a Picture and Scaling Up*

A. The Big Ideas

Purpose of This Lesson

This lesson provides students with another example of scale factor — one that they can easily see has a real-world application.

Mathematical and Problem-Solving Goals

- Use the concepts of similar figures to enlarge a picture.
- Determine the scale factor needed to enlarge a picture to a specific size.
- Use scale factors to find the lengths of corresponding sides.
- Find a missing measure in a pair of similar figures.

Connections to NCTM Standards

- Determine an appropriate scale and use scale drawings or models in applications.
- Use ratios and proportions to solve problems involving scale factors.
- Work flexibly with equivalent fractions, decimals and percents; compare and order these numbers efficiently and accurately; find their approximate location on a number line; and choose appropriate and convenient forms of these numbers for solving problems.
- Recognize, use and learn about mathematics in contexts outside of mathematics.

B. The Lesson

Recommended Mathematical Background

- Familiarity with the concept of scale factor (from Lesson 2)
- Measuring in inches or centimeters
- Some familiarity with fractions and decimals

Materials

- Rulers
- Small photographs
- Large sheet of dot paper
- Calculators
- Large sheet of chart paper

Time

- 60–80 minutes (includes time for each group to present)

**This extension lesson and the one that follows appear in the classroom video in Proportionality & Similar Figures, Workshop 2: In Practice.*

TEACHING TOOLS

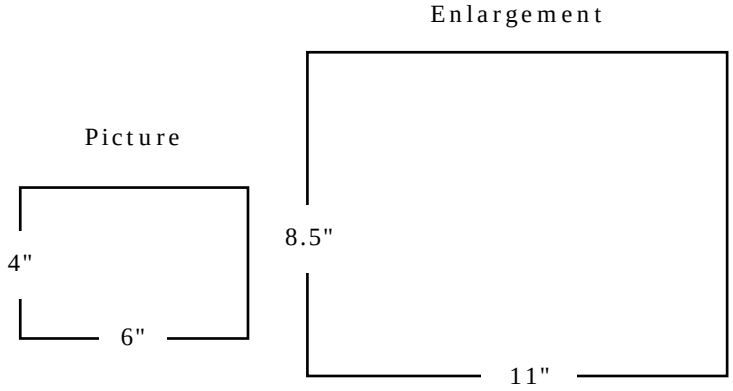
- As you discuss and use this lesson, make sure to take advantage of the Teacher Planning Tools (pages 151-157):
  - Planning a Math Unit: Launch-Explore-Summarize Teaching Model
  - Lesson Planner Template
  - Questions to Stimulate Student Thinking
  - Guidelines for Grouping.
- Refer to Why This Topic Matters (page 37) to help make this topic relevant to your students.
- Check out our Web site (www.learner.org/channel/workshops/missinglink) for additional tools and resources — and to join The Missing Link online discussion forum.

Lesson Overview

In this investigation, students use the concept of scale factor to make an enlarged copy of a small photograph. You may use any old photographs or pictures from magazines or newspapers for this activity. Try to cut the pictures so that there is a variety of sizes (3" x 5", 2.5" x 4", etc.). When given the photograph, students determine the dimensions of an enlargement that is similar to the original and is the largest picture that would still fit on an 8 1/2" x 11" sheet of paper. Students find the largest scale factor that can be used and determine if the new photograph would fill the paper exactly. If it doesn't, they calculate how much must be trimmed off either the length or the width of the paper. They make sketches of their new enlargement and explain all of their work.

Example

Suppose your original photograph measures 4" x 6". What would be the dimensions of the picture if you were to make the largest enlargement that would still fit on an 8.5" x 11" sheet of paper?



4 x _____ = 8.5" Scale factor is 2.125 (8.5 ÷ 4).

Therefore, 6 x 2.125 = 12.75" *(the length of the side of a similar rectangle).*

So, one enlargement similar to the original photo would measure 8.5" x 12.75". Since the paper measures only 8.5" by 11", this enlargement is impossible.

So let's look at scaling the 6" side up to the 11" side.

6 x _____ = 11" Scale factor is 1.83 (11 ÷ 6).
4 x 1.83 = 7.32" *(the length of the side of a similar rectangle).*

So, the largest enlargement that could be made measures approximately 7.32" x 11", using a scale factor of 1.83, and 1.18" would need to be trimmed off the width for the enlargement to remain similar to the original picture.

ENLARGING A PICTURE AND SCALING UP

The concept of similarity has many practical applications. For example, architects and designers often make a model of an object and then scale it up or down to make the real object. Photographers take pictures and then need to make enlargements for their customers.

Take a small photograph and enlarge it (scale up) so that it completely fills an 8.5" x 11" piece of paper. The enlargement must be kept similar to the original.

- Is this possible? Explain. Be sure to show your calculations and explain your solution in detail. Tell what you did and why you did it.
- If the picture cannot be scaled up to fit the paper exactly, should the length or width of the paper be trimmed when you make the largest enlargement possible?
- How much should be trimmed off the length? How much should be trimmed off the width? Explain and show your work.

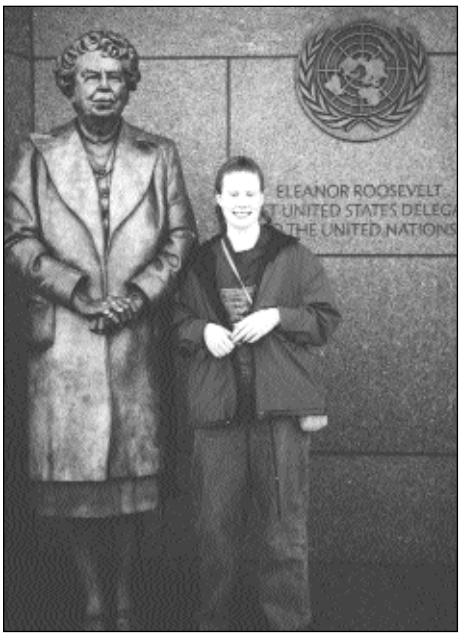
Source: Adapted from *Connected Mathematics Project – Stretching and Shrinking*

■ Extension 2: The Carly Problem

Give students photographs that have a person holding a piece of paper or standing next to an object with known dimensions. Students determine the height of the person in the photo by using scale factors and proportionality.

For example, while on vacation with her family in Washington, D.C., Carly had her picture taken in several places. Use all four photos and the information given below each picture to determine Carly's height. Your answer may be different for each picture. Decide on one final answer and be sure to support your choice.

- Your work should be neat and easy to follow.
- Clearly explain what you did and why.
- Support your final answer with the mathematics you have shown and the explanation you have written.



The statue of Eleanor Roosevelt is seven feet tall. Find Carly's height.

The bronze statue of the Roosevelt dog is three feet tall from the tip of his ears to his front paws. Find Carly's height.

Carly is holding a picture that measures 8.5" x 11". Find Carly's height.

Carly is holding a picture that measures 8.5" x 11". How tall is Carly?