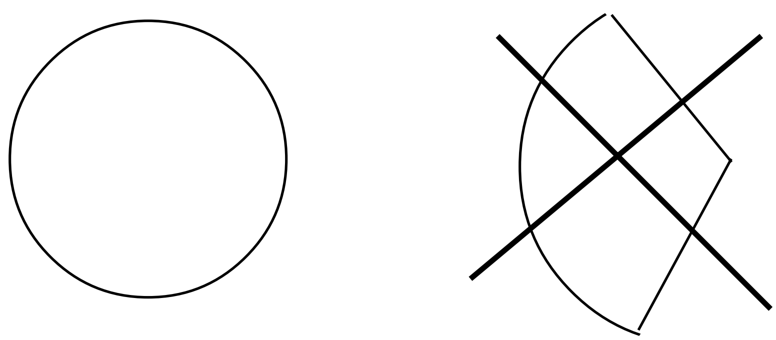
Take One-Third

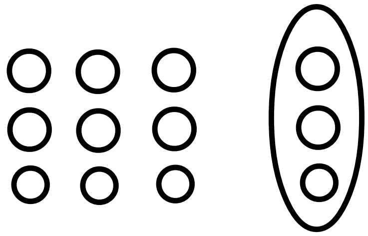
As my seventh and eighth graders entered our classroom and found their seats, their attention turned to the “starter problem” written on the overhead. I asked them to work on it silently in preparation for the day’s lesson on multiplying fractions.

On your own, draw a picture where you take $^1/_3$ of $1\,^1/_3$. Hint: Start with a picture of $1\,^1/_3$.

I finished administrative tasks as the students worked, then walked around to look at their pictures. I decided to ask Linda and Bob to show their solutions on the board, so that I could illustrate the use of both continuous and discrete fractions. We turned first to Linda’s picture.



“How did you start the problem, Linda?”
“I just drew 1 and $^1/_3$,” she said.
“So this circle represents a whole, and this piece is $^1/_3$ of another equal-sized whole?”
“Yeah.”
Several students commented that they had drawn very similar pictures. I asked Linda to explain how she solved the problem.
“I just took away $^1/_3$ from 1 and $^1/_3$,” she answered, as she crossed out the $^1/_3$.
“Listen to what you just said,” I prompted.
“I just took away $^1/_3$ from 1 and $^1/_3$,” Linda insisted.
“What operation did you say out loud?” I asked.
“Take away — subtraction.”
When I directed her attention back to the problem on the overhead, she looked confused, saying: “Take $^1/_3$ of $1\,^1/_3$. I don’t get it. This is weird.”
Still hoping the class would be able to discover the proper procedure on its own, I switched to Bob, expecting that his solution would be both correct and easier for the class to understand.



He explained how he started. “I thought of 9 spots being in the whole and then 3 more would be $\frac{1}{3}$.”

“So how did you find $\frac{1}{3}$ of $1\frac{1}{3}$?”

“I just took $\frac{1}{3}$,” he replied indicating three of the spots.

“You should take 4,” Jim and a few other students cried out. I asked them to think about what it means to take $\frac{1}{3}$, giving a hint by pointing to the denominator.

“Divide them into 3 equal groups,” Amanda volunteered, “and you get 1 of those groups or $\frac{1}{3}$.” Then I attempted a real life example that would relate Bob’s problem to the previous day’s lesson. I asked them to imagine that Bob’s items were popsicles and to think about $\frac{1}{3}$ of all of them. Bob could get one part, his brother another equal part, and his mom another third. His dad is on a diet. How many popsicles would it be fair for Bob to eat? Jane and several students said 4, a few said 3. I called on Jane to explain.

“$\frac{1}{3}$ of 12 is 4. You just divide,” she replied.

“So we weren’t supposed to take away $\frac{1}{3}$. These are division problems,” Max realized. “Why didn’t you tell us?” The whole class was a little unhappy with my “deception.”

All this took place in a general math class in a medium-sized suburban middle school with 850 students. There were twenty seventh graders and six eighth graders in the class. I had chosen fractions as one of my teacher-evaluation curricular areas for the year.

I had begun the unit by giving a pretest on basic fraction concepts and operations, including some word problems. Because the scores were on average very low, I decided to spend a couple of months working with these ideas so my students would show substantial growth on the post-test.

We spent the first few weeks developing meaning for fractions, placing a strong emphasis on being able to draw a picture of a fraction amount. Most pictures showed subdivisions in rectangles, with number lines being used occasionally. I offered examples of work with discrete fractions, such as $\frac{12}{24}$ meaning 12 of the 24 original pieces of candy in a box, which in turn means the box is half full. By the time we reached the current lesson, the students also had some experience drawing representations of subtraction problems such as:

$$\frac{1}{2} - \frac{3}{8}$$

On the day before this lesson the students had worked on pictures that introduced the concept of fraction multiplication. They had sketched problems such as $\frac{1}{3}$ of 27 pieces of gum and had figured $\frac{1}{4}$ of 16 candies. The word *multiplication* had not been formally used at this point.

So here we were trying to begin a lesson on multiplying fractions, and now Linda’s work had revealed that she thought we were talking about subtraction. Then, from Bob’s example the class had impulsively concluded that “you just divide.” I wondered if it had been a mistake to follow my usual custom of having students experiment with a new concept on their own before the formal lesson. Was it practical—or possible—to capitalize on these misunderstandings and proceed with the lesson? Or should I have started over and approached the lesson by giving a little more guidance?

I now see that similarities in the language of multiplying and subtracting fractions call for a careful choice of words. But beyond that, my own understanding of fractions has been shaken Max’s statement. It seems that you do divide when multiplying fractions. But how am I going to make sense of that to my students?