MATHematics illuminated

PARTICIPANT GUIDE

UNIT 7

UNIT 07

MAKING SENSE OF RANDOMNESS

PARTICIPANT GUIDE

ACTIVITIES

NOTE: At many points in the activities for Mathematics Illuminated, workshop participants will be asked to explain, either verbally or in written form, the process they use to answer the questions posed in the activities. This serves two purposes: for the participant as a student, it helps to solidify any previously unfamiliar concepts that are addressed; for the participant as a teacher, it helps to develop the skill of teaching students "why," not just "how," when it comes to confronting mathematical challenges.

ACTIVITY 1

The following questions visit some of the important topics from Unit 7 in more depth.

A

1. Suppose that 100 people are to flip a fair coin simultaneously. Suppose further that the 100 people decide to form a traveling show, traveling from city to city each day and performing their simultaneous coin flip once every night in front of an enthralled audience. After a year, what would you expect the average number of flips that end up "heads" to be (per night)?

2. What is the probability that after 100,000 shows, at least one of the shows involves 100 simultaneous flips that all come up "heads?"

3. Suppose that there are about 10^{24} grains of sand on Earth. What is more likely, flipping 100 heads in a row, or randomly selecting a specific grain of sand on Earth? Compare the two probabilities.

B

1. Say that the average height of an American male is 5' 10", and the average height of an American female is 5' 5". Furthermore, say that the standard deviation for men is 3", and for women it is 2.5". Find the normalized heights for the people in your group. Express your answers in terms of standard deviations.

2. Compare your normalized height to the normalized heights of the following famous people:

Yao Ming, U.S.A.(Professional Basketball Player): 7' 6"

Ekatarina Gamova, (Russian volleyball player and one of the tallest female athletes as of 2007): 6' 9"

Leonid Stadnyk, Ukraine (Tallest living man as of 2007): 8' 5"

De-Fen Yao, China (Tallest living woman in the world as of 2007): 7' 8"

Robert Wadlow, U.S.A. (Tallest recorded human as of 2007): 8' 11"

3. Discuss the various assumptions behind this activity.

ACTIVITY 1

C

Suppose that a lottery has the following payoff structure:

Outcome (selected values) and wildcard	Prize	Odds
○○○○○ + ●	$120,000,000.00	1 in 146,107,962.00
○○○○○	$200,000.00	1 in 3,563, 608.83
○○○○ + ●	$10,000.00	1 in 584,431.85
○○○○	$100.00	1 in 14,254.44
○○○ + ●	$100.00	1 in 11,927.18
○○○	$7.00	1 in 290.91
○○ + ●	$7.00	1 in 745.45
○ + ●	$4.00	1 in 126.88
●	$3.00	1 in 68.96

1. What is the expected value of a ticket in this lottery if the Grand Prize is $120,000,000?

D

Suppose that you must select a three-student team to send to the state math competition. Out of the 12 students in your math club, how many possible teams can you select? Assuming that each student is equally likely to be selected, what is the probability of a particular student being selected for the team?

E

In the 1880s, about two out of every thousand couples got divorced each year. In the 1950s it was about 11 out of every thousand. In the 1980s, it was about 22 out of every thousand. Find the probability that a couple would be together after 30 years for each of the time periods. Don't worry about the probability changing each year.

ACTIVITY 2

In this activity you will explore some of the many uses of the Central Limit Theorem (CLT), one of the most important concepts in probability and statistics. The CLT was described in the text as having to do with the distribution of results of sets of many trials. It said that even if the results of a single event are not normally distributed, the results of multiple trials of the single event will be, at least approximately.

For example, while the outcome of a single coin toss is certainly not normally distributed, the distribution of outcomes of sets of many coin tosses is (approximately). The advantage conferred by knowing that something is approximately normally distributed is that one can then use the concepts of the mean and standard deviation to make predictions about large populations. We'll see how this is done in the following activity.

A

THE DISHONEST COIN

1. Suppose that you have come into possession of a coin that preferentially lands "heads" up with 0.8 probability. If you were to flip this coin 100 times, how many "heads" would you expect, on average? Call this result the "mean."

2. The outcome of any particular set of 100 flips is by no means guaranteed to be what you just calculated. You can use the CLT, however, to make predictions about how close the results of a specific set of 100 flips will be to the mean. To do this, we first need to find the standard deviation, which is the average number of flips by which a given set of 100 flips should differ from the mean. If the standard deviation is defined as:

$$\sigma = \sqrt{(\text{\# of flips}) \times (\text{probability of heads}) \times (\text{probability of tails})}$$

what is the standard deviation for sets of 100 flips of your "dishonest" coin?

3. Using the 68-95-99.7 rule from the text (the probability that a given trial falls within 1, 2, or 3 standard deviations of the mean, respectively), if you perform 100 coin flips of your dishonest coin, what is the probability that between 84 and 76 of the results will be heads?

4. Give the range in which you expect the number of heads to fall 99.7% of the time.

ACTIVITY 2

5. If you perform 1,000 sets of 100 flips, how many would you expect to have more than 92 or fewer than 68 "heads"?

B

Suppose that you work for a company that makes frozen hamburger patties. One of your beef suppliers reports that all of his cows have been diagnosed with mad cow disease, and he is afraid that his most recent shipment of cattle to you was infected as well. This supplier's cattle make up 1 out of every 10 of the cattle you process into hamburger patties. Because you process cattle from each supplier separately into hamburger patties, yet mix patties randomly from different suppliers when packaging, potentially 1 out of every 10 of your frozen hamburger patties is contaminated.

1. If you package patties in bags of 50, how many patties per package are, on average, contaminated?

2. What is the standard deviation for bags of 50 patties, 1 in 10 of which are contaminated?

3. If you shipped 10,000 bags of 50 patties, how many bags would you expect to have between 3 and 7 contaminated patties?

4. How many bags would have between 0 and 12 contaminated patties? How many bags would fall outside of this range?

5. What are the chances that a bag contains no contaminated patties?

6. Using the probability you just found, how many uncontaminated bags would you expect to find out of 10,000?

7. How would the above result change if 1 out of 100 patties were contaminated instead of 1 out of 10?

8. Say that you ship 100 shipments of 10,000 bags, with 1 out of 100 patties contaminated. Give the range of expected numbers of safe bags per shipment for 99 of the shipments.

9. How many shipments would you have to send before you would expect to ship a completely uncontaminated batch? (feel free to approximate liberally)

ACTIVITY 2

C

Say that you are conducting an exit poll for an election. You interview an appropriate (unbiased) sample of 1,200 people as they leave the voting area and ask them who they voted for. You find that 54% of people say they voted for candidate A and 46% say they voted for candidate B. You can view each person polled as a coin that comes up heads 54% of the time and tails 46% of the time.

1. What is the standard deviation of your sample?

2. Express this deviation as a percent of the sample size.

The percent you just found (actually, twice the percent you just found) is what is commonly known as the "margin of error" in polling. It involves the assumption that the standard deviation of your sample is very close to the standard deviation of the pool of all people voting, which is usually a reasonable assumption.

3. How likely is it that candidate A garners between 52.6% and 55.4% of the overall vote?

4. How likely is it that candidate A garners between 51.2% and 56.8% of the overall vote?

5. What is the chance that candidate A won the overall election outright? Assume that a candidate needs 51% of the vote to win outright. Assume f urther that the difference in percent certainty between the 2nd and 3rd standard deviation (the difference between 95% and 99.7% probability) can be apportioned evenly with portions of a whole standard deviation—that is, half of a standard deviation yields a percent certainty halfway between 95% and 99.7%. (Note: because the normal distribution is a curve, this isn't actually the case, but for a rough estimate, it will work.)

6. If you did the same poll of 1200 people for 100 elections and got 54% for candidate A in every one of them, how many of those elections would you expect ending up going to candidate B?

7. How does this make you feel about the reliability of exit poll data?

ACTIVITY 3

A

Imagine that you are about to start a blog and wish to drive as much traffic to it as possible. You have $10,000 set aside for marketing. You know that a friend of yours recently started a blog and was able to attract 100,000 visitors after spending only $5,000 on marketing. She did this by having a lottery for people who post comments on the site.

1. Your friend set up an account with a large search engine company to place ads for her blog on other websites. She agreed to pay $0.02 per visitor that the ads attracted to her site, up to 100,000 visitors. How much of her marketing budget did she set aside for the search engine?

In order to make her ads stand out, she wanted to make a lottery that sounded as attractive as possible. She came up with the following possible ads:

a) "Post a comment on Sara's blog and be automatically entered in a drawing for a Gizmophone2000."

b) "Post a comment on Sara's blog and be automatically entered in a drawing for one of 10 Gizmophone2000 Nanos."

c) "Post a comment on Sara's blog and be automatically entered in a drawing for a Gizmophone2000 Nano, or one of 1,000 other prizes.

2. If Sara's search engine company requires her ad to state the odds of winning a prize, and she will not administer the lottery until she has 100,000 entries, which of the above three ads should she pick as the most likely to attract someone's mouseclick? Justify your answer.

3. Find the expected value of each version of the promotion.

4. Suppose that Sara goes with choice "c"; the actual breakdown of prizes is as follows:

1 × Gizmophone2000 Nano
2 × $100 gift certificates to iTones.com
200 × $5 gift certificates to iTones.com
798 × Sara's blog keychain flashlights (valued at $1.87 each)

ACTIVITY 3

What are the odds of winning something worth $100 or more?

5. Sara estimates that if she draws 100,000 visitors to her blog during this promotion, at least 10% will continue to visit the site once a week. Each of these visits (both during the promotion and during the month) counts as a "hit." She estimates that out of every 100 hits, 50 visitors will click on the ads of her sponsors, for which she is paid $0.03 each. She further estimates that one visitor out of 100 will buy a Sara's blog t-shirt, for which she nets $5. How much net income can she expect as a direct result of her promotion in her first 6 months if she starts the lottery promotion at the beginning of the first month? (We are concerned only with the direct effects of the promotion, so ignore any new visitors and website costs.

B

As a group, design a similar promotion for your own blog. Remember that you have $10,000 for marketing; suppose that your goal is to attract 500,000 visits as a result of the promotion. Feel free to choose how much you spend on search engine ads, what prizes you will offer, and how you will structure your prize scheme. Include the expected value of a typical entry. Your promotion should consist of a prize structure, the associated probabilities, the expected value of an entry, and the text for an attractive web ad.

CONCLUSION

HOW TO RELATE TOPICS IN THIS UNIT TO STATE OR NATIONAL STANDARDS

DISCUSSION

Mathematics Illuminated gives an overview of what students can expect when they leave the study of secondary mathematics and continue on into college. While the specific topics may not be applicable to state or national standards as a whole, there are many connections that can be made to the ideas that your tstudents wrestle with in both middle school and high school math. For example, in Unit 12, In Sync, the relationship between slope and calculus is discussed.

Please take some time with your group to brainstorm how ideas from Unit 7, Making Sense of Randomness could be related and brought into your classroom.

Questions to consider:

Which parts of this unit seem accessible to my students with no "frontloading?"

Which parts would be interesting, but might require some amount of preparation?

Which parts seem as if they would be overwhelming or intimidating to students?

How does the material in this unit compare to state or national standards? Are there any overlaps?

How might certain ideas from this unit be modified to be relevant to your curriculum?

WATCH VIDEO FOR NEXT CLASS

Please use the last 30 minutes of class to watch the video for the next unit: Unit 8, Geometries Beyond Euclid. Workshop participants are expected to read the accompanying text for Unit 8, Geometries Beyond Euclid before the next session.

NOTES