Session 3

The Metric System

Key Terms in This Session

New in This Session

- metric system
- prefixes
- referents
- U.S. customary system

Introduction

There are two measurement systems used in the United States: the English or U.S. customary system and the metric system. A measurement system consists of units, which are defined in specific ways, and their symbols. Within a system, there are relationships between units, such as between inches and feet in the English system or centimeters and meters in the metric system. In a coherent measuring system, a small number of independent units fit together in simple ways, and all other units are defined in terms of the base units. The metric system is almost completely coherent, whereas the English system is not coherent at all! In this session, you will learn about the metric system. **[See Note 1]**

For information on required and/or optional materials for this session, **see Note 2.**

Learning Objectives

In this session, you will learn to do the following:

- Understand the relationships between units within the metric system
- Represent quantities using different units
- Estimate and measure quantities of length, mass, and capacity, and solve measurement problems
- Understand the basic measurement ideas that lead to accurate measurement

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Note 1. Most Americans are not very comfortable using the metric system. We can't estimate with any accuracy the length of a room in meters, or approximate a kilogram of potatoes. Yet we live in a world that is metric; the United States is one of the few nations that has not converted to the exclusive use of the metric system.

This session introduces the key features of the metric system. We examine the relationships between units within the metric system and learn how to represent quantities using different units. You will have an opportunity to develop metric benchmarks by estimating and measuring quantities of length, mass, and capacity.

Remember, children who frequently use materials based on the metric system (for example, metersticks and centimeter cubes) will be able to visualize meters, centimeters, grams, and so on, and the relationships between certain units.

Note 2. Materials Needed:

- Chalk
- Empty plastic 1 L, 2 L, and 3 L bottles

Additional materials needed:

- Metric tape measure and ruler
- 12 metersticks
- Metric trundle wheels (optional)
- Metric graduated beakers (500 mL or 1,000 mL)
- A variety of metric scales: two-pan balance scale, spring scale, and a personal scale (optional)
- Metric weights (1 g, 5 g, 10 g, 25 g, 100 g, 500 g, 1 kg) (you could also use centimeter snap-together cubes that weigh 1 g each) (optional)

The above materials can be purchased from:

ETA/Cuisenaire, 500 Greenview Court, Vernon Hills, IL 60061
Phone: 800-445-5985/800-816-5050 (Customer service)
Fax: 800-875-9643/847-816-5066 • http://www.etacuisenaire.com

Part A: Metric System Basics (35 min.)

History

The metric system was introduced in France in the 1790s as a single, universally accepted system of measurement. But it wasn't until 1875 that the multinational Treaty of the Meter was signed (by the United States and other countries), creating two groups: the International Bureau of Weights and Measures and the international General Conference on Weights and Measures. The purpose of these groups was to supervise the use of the metric system in accordance with the latest pertinent scientific developments.

Since then, the Bureau and Conference have recommended improvements in terms of the accuracy and reproducibility of metric units. In fact, by 1960, improvements were so great that a "new" metric system, called the International System of Units (abbreviated SI), was created. The metric system we use today is actually SI. Although there are some discrepancies between the two systems, these differences are slight from a nonscientific perspective; therefore, we will refer to SI as "the metric system."

Let's begin by reviewing the metric system. **[See Note 3]**

Problem A1. Make a list of some of the facts, relationships, and symbols you recall about the metric system.

Problem A2. Why do you think most countries use the metric system?

Units and Prefixes

One of the strengths of the metric system is that it has only one unit for each type of measurement. Other units are defined as simple products or quotients of these base units.

For example, the base unit for length (or distance) is the meter (m). Other units for length are described in terms of their relationship to a meter: A kilometer (km) is 1,000 m; a centimeter (cm) is 0.01 of a meter; and a millimeter (mm) is 0.001 of a meter.

Prefixes in the metric system are short names or letter symbols for numbers that are attached to the front of the base unit as a multiplying factor. A unit with a prefix attached is called a multiple of the unit—it is not a separate unit. For example, just as you would not consider 1,000 in. a different unit from inches, a kilometer, which means 1,000 m, is not a different unit from meters.

Try It Online! **www.learner.org**

This activity can be explored as an Interactive Activity. To find the activity, go to the *Measurement* Web site at www.learner.org/learningmath and find Session 3, Part A.

Use the chart below to explore some common units and prefixes in the metric system. Pay attention to the patterns that emerge as you look at the different prefixes.

Height	Volume	Weight
skyscraper **0.35** kilometers **3.5** hectometers **35** dekameters **350** meters **3500** decimeters **35000** centimeters **350000** millimeters	**bottle** **0.002** kiloliters **0.02** hectoliters **0.2** dekaliters **2** liters **20** deciliters **200** centiliters **2000** milliliters	**cat** **5** kilograms **50** hectograms **500** dekagrams **5000** grams **50000** decigrams **500000** centigrams **5000000** milligrams
film **0.000035** kilometers **0.00035** hectometers **0.0035** dekameters **0.035** meters **0.35** decimeters **3.5** centimeters **35** millimeters	**medicine** **0.00002** kiloliters **0.0002** hectoliters **0.002** dekaliters **0.02** liters **0.2** deciliters **2** centiliters **20** milliliters	**cherries** **0.005** kilograms **0.05** hectograms **0.5** dekagrams **5** grams **50** decigrams **500** centigrams **5000** milligrams

Note 3. This session focuses only on the metric system, but you may also want to use this session to review what you know about the customary system. The concept of a coherent system may be confusing. If the customary system were coherent (which it is not), then there would be a single base unit for length; area and volume would also be based on this unit. For example, if the inch were the base unit, then we would measure area in square inches (not in acres or square yards) and volume in cubic inches (not in pints or cubic feet). In a coherent system, units can be manipulated with simple algebra rather than by remembering complex conversion factors.

Part A, cont'd.

Working With Metric Prefixes

Here is a detailed table of some metric prefixes: **[See Note 4]**

Prefixes and Their Equivalents

Symbol	Prefix	Factor	Ordinary Notation	U.S. Name	European Name (if different)
T	tera	10^{12}	1,000,000,000,000	trillion	billion
G	giga	10^{9}	1,000,000,000	billion	milliard
M	mega	10^{6}	1,000,000	million	
k	kilo	10^{3}	1,000	thousand	
h	hecto	10^{2}	100	hundred	
da	deka	10^{1}	10	ten	
		10^{0}	1	one	
d	deci	10^{-1}	0.1	tenth	
c	centi	10^{-2}	0.01	hundredth	
m	milli	10^{-3}	0.001	thousandth	
μ	micro	10^{-6}	0.000 001	millionth	
n	nano	10^{-9}	0.000 000 001	billionth	thousand millionth
p	pico	10^{-12}	0.000 000 000 001	trillionth	billionth

Problem A3.

a. What patterns do you notice in this table?

b. Some of these prefixes are more commonly used than others, particularly when the base unit is a meter. Which prefixes are you most familiar with?

c. How can a centimeter, a millimeter, a kilometer, and a micrometer be expressed in terms of a meter?

d. A kilometer is 1,000 m, but we can also state that it is 1,000,000 mm. Explain how these relationships work.

e. Try expressing the following quantities in at least two different ways: centimeter, millimeter, and decimeter.

Problem A4. You have probably noticed that the metric system never uses fractions. Instead, fractional quantities are recorded using decimals. Why do you think this is the case? **[See Tip A4, page 63]**

Problem A5.

a. If you want to change 3,600 m to kilometers, what do you do? Explain.

b. If you want to change 0.028 m to millimeters, what do you do?

c. If you want to change 4,600,000 mm to kilometers, what do you do?

Take It Further

Problem A6. The Sun is 150 gigameters (Gm) from the Earth. The Moon is 384 megameters (Mm) from the Earth. How many times farther from the Earth is the Sun than the Moon?

Note 4. If you are working in a group that isn't very familiar with the metric system, you can take this opportunity to practice expressing quantities in more than one way. For example, working in pairs or as a class, participants can make up questions for one another to answer and/or quantities to express using different prefixes. Refer to Problems A3–A5 for guidance.

Part B: Metric Units (85 min.)

Length

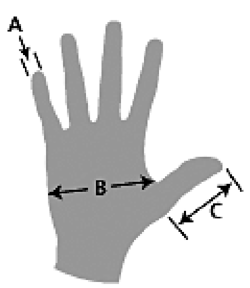
As we've seen, the base unit for length (or distance) is the meter. Meter comes from the Greek word "metron," which means "measure."

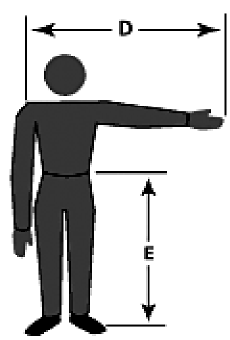
Many of us do not have a strong intuitive sense of metric lengths, which may be a result in part of our limited experience with metric measures and estimates. It is, however, important to have referents for measures, as referents make measurement tasks easier to interpret and provide us with benchmarks against which to test the reasonableness of our measures. **[See Note 5]**

Problem B1. Take the centimeter grid on page 62 and paste it onto stiff cardboard paper. Cut and tape pieces together to build a meterstick, and explore how you would mark decimeters, centimeters, and millimeters on it.

Problem B2.

a. Find a friend or colleague, and use a metric tape measure to measure the following body lengths: A, B, C, D, and E (as pictured below). Your goal is to try to find your own personal referents for 1 cm, 1 dm, and 1 m.





b. Using the information you gathered, estimate these lengths:
   - The height of a door
   - The length of your table
   - The width of a notebook
   - The thickness of a dime
   - The length, width, and height of the room

Take It Further

Problem B3.

a. Approximate a distance of 100 m. First plan how you will determine this length, and then measure this distance outside. Mark off 100 m using chalk, and then use a trundle wheel to check your approximation. (A trundle wheel is a plastic wheel, usually graduated in 5 cm intervals, designed to measure lengths by counting the number of clicks, each of which equals 1m.)

b. Use this distance to estimate the time it would take you to walk 1 km. If it's a nice day, check your estimate by actually walking 1 km. What is your average walking pace? **[See Note 6]**

Note 5. When measuring objects using the metric system, it is important to establish benchmarks for common lengths, such as meter, decimeter, and centimeter. In addition, you should actually make the measurements and compare your estimates to your measurement data. Reconciling the differences between your estimates and measures will help you improve your ability to make reasonable estimates using the metric system.

Note 6. Be aware that errors in approximating 100 m are going to be compounded tenfold when using your 100 m distance to approximate 1 km (since 100 • 10 = 1,000 m). You may want to check the distance using a car or bicycle odometer. When you know the approximate time it takes you to walk 1 km on flat terrain, then you can use time to estimate distances (e.g., I walked for 20 minutes, so I know I have traveled about 2 km).

Part B, cont'd.

Video Segment (approximate time: 10:00–12:10): You can find this segment on the session video approximately 10 minutes after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

In this video segment, Mary and Susan work together to establish some referents for measuring lengths using their own bodies, for example, the width of a hand or an arm length. They use those referents to make measurements and then compare them to standard-unit measurements.

Why is it important to establish such referents for measuring? Can you think of any situations in which they might be useful?

Liquid Volume

Measures of liquid volume, sometimes referred to as capacity, include the liter (L) and the milliliter (mL). These terms are holdovers from an older version of the metric system and, because they are so well known, are approved for use with the current SI. Volume, whether liquid or solid, is a measure of space. Solid volume is measured using cubic meters (m^3) as the base unit. Liquid volume is most often measured using liters. In Session 8, we will explore measures of solid volume in detail, but we will begin to examine the relationships among measures of solid and liquid volume in this session.

By definition, a liter is equivalent to 1,000 cm^3 (or 1 dm^3). This leads to the conclusion that 1 mL is equivalent to 1 cm^3. Large volumes may be stated in liters but are usually recorded in cubic meters. **[See Note 7]**

Problem B4.

a. Use metersticks and masking tape to construct a cubic meter. The metersticks will form the edges of the cubic meter. **[See Note 8]**

b. Make a list of measures equivalent to 1 m^3 (for example, using cubic decimeters, cubic centimeters, cubic millimeters, and liters).

Note 7. Whereas the base unit for volume is the cubic meter, most practical day-to-day situations find us determining the capacity of smaller containers, and thus cubic centimeters or cubic millimeters might also be used. The relationship between cubic centimeters and milliliters (1 cm^3 = 1 mL) and between cubic decimeters and liters (1 dm^3 = 1 L) is an important one to establish. Models can help people visualize these relationships. If you have metric base ten blocks, then the "small units cube" (1 cm^3) is equivalent to 1 mL, and the "thousands cube" is equivalent to 1 dm^3; this cube, if hollow, will hold 1 L. Compare a milliliter and a cubic centimeter as well as a liter and a cubic decimeter. If possible, pour 1 L of water into a hollow decimeter cube.

Note 8. If you are working in a group, it is worth the time and effort to have groups of four people construct a cubic meter, using metersticks as the edges of the large cube. If supplies are limited, make 1 m^3 as a model for the whole group to observe. Participants can hold the metersticks in place or tape them together.

Some people may have difficulty listing the equivalent measures for 1 m^3. A common error is to think that there are 100 cm^3 in 1 m^3. Use the model to show that this amount is much too small. Try listing an equivalent measure for each dimension before finding the volume. For example, since 100 cm = 1 m, the length, width, and height of the cube are all 100 cm, and you can find the volume by multiplying length times width times height.

Part B, cont'd.

Problem B5. Estimate the capacity of the following in liters or milliliters:

- A teacup
- A thimble
- A car's gas tank

Problem B6. Sometimes in medical situations, we hear of someone receiving an injection of 3 cc of medicine. What do you think this measure, 3 cc, represents?

Problem B7. Examine a 1 L bottle. In addition to the liquid, there is some air space in the bottle. So what is a liter—the amount of liquid, or the entire volume of the bottle? Find out by pouring the liquid into a graduated 1 L or 500 mL container. Try this with a 2 L or 3 L bottle as well. How much liquid is actually in a 1 L bottle? In a 2 L bottle? **[See Note 9]**

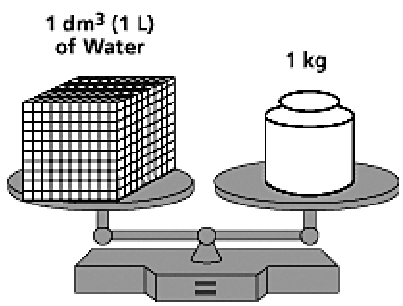
Problem B8. The average woman has a lung capacity of about 4.4 L, and the average man has a lung capacity of about 5.8 L. What is their lung capacity in cubic decimeters? Compare the two units.

Mass

Whereas weight measures the gravitational force that is exerted on an object, mass measures how much of something there is; thus, mass is closely related to volume. The weight of an object can change depending on its location (e.g., on the Earth or on the Moon), but the mass of the object (how much of it there is) always stays the same.

Mass and weight are often confused, because our two systems of measurement use different terms. In the metric system, kilograms and grams are measures of mass, but in the U.S. customary system, ounces and pounds are measures of weight. When using the metric system, we should really state that we are measuring mass, saying, for example, "I have a mass of 60 kg" rather than "I weigh 60 kg," but this goes against convention. Throughout this course, we will use both terms (but regardless of the term we use, mass is what we'll be finding!). **[See Note 10]**

The base unit of mass is the kilogram (kg). In the 1790s, a kilogram was defined as the mass of 1 L (cubic decimeter, or dm^3) of water:



Though that definition has changed somewhat with time, here is a definition that is close enough for ordinary purposes: There are 1,000 g in 1 kg, and 1,000 g occupy a volume of 1,000 cm^3, or 1 L. Therefore, 1 g of water weighs the same as 1 cm^3 of water and occupies 1 mL of space. In other words, for water:

$$1{,}000 \text{ g} = 1 \text{ kg} = 1{,}000 \text{ cm}^3 = 1 \text{ dm}^3 = 1 \text{ L}$$

and

$$1 \text{ g} = 1 \text{ cm}^3 = 1 \text{ mL}$$

Note 9. Since you will be pouring liquids in and out of bottles to find the capacity, this can get a bit messy. But to fully understand the size of metric units, it is important to actually do the measurements. If there is time, test more than one type and brand of liquid. Is there the same amount of liquid in a 1 L bottle of soda as in a 1 L bottle of water? Or test many 1 L bottles of the same brand. Is the capacity consistent from one bottle to the next? Be sure to discuss or reflect on your findings.

Note 10. If you are working in a group and you did not discuss the difference between mass and weight during Session 1, do so now. Have everyone explain in their own words how mass and weight differ. Ideally you want to have available a variety of scales: pan balances, three-arm balances, spring scales, and a metric bathroom scale.

Part B, cont'd.

Kilograms are used to weigh just about everything but very light objects (which are weighed in grams) and very heavy objects (which are weighed using metric tons). A gram is almost exactly the weight of a dollar bill. A metric ton is equivalent to 1,000 kg (so it can also be thought of as a megagram) and should not be confused with the common American ton in the U.S. customary system. In fact, the metric ton is often referred to by its French and German name, tonne, to distinguish it as a metric measure. Most cars have a mass of between 1 and 2 tonnes; a large diesel freight locomotive has a mass of approximately 165 tonnes.

As with metric lengths, it is useful to establish benchmarks for metric mass measures.

Problem B9.

a. Using the centimeter grid paper from page 62 build a cubic decimeter. What is its capacity? What is its weight if filled with water?

b. What is the capacity and weight of 1 cm^3?

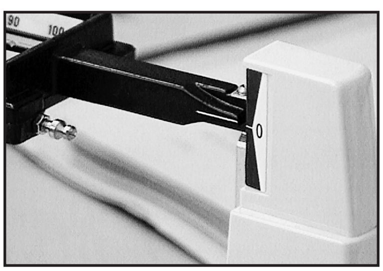
[See Tip B9, page 63]

Problem B10. Some people use balance scales like the one illustrated above, and some people use spring scales like a typical grocery or bathroom scale. How are the two different? **[See Tip B10, page 63]**

Problem B11. Look around the room you're in now, and find one or more objects that you estimate has each of the masses listed below. If a scale is available, use it to measure each object to corroborate your estimates. If a scale is not available, select some food products (with very light wrappers) such as candy bars and cereal. Estimate their mass and compare your estimate to the mass indicated on the product. **[See Note 11]**

a. 1 g   b. 100 g   c. 500 g   d. 1 kg

Problem B12. Take an empty plastic liter bottle and weigh it. Then fill the bottle with very cold water and weigh it again. What do you notice about the weight of the liter of water? Explain your findings.



Video Segment (approximate time: 19:20–21:15): You can find this segment on the session video approximately 19 minutes and 20 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

Watch the participants as they measure the mass of a 1 L bottle filled with water. They are surprised to discover that it's a little more than 1 kg. This is not what they expected, so they contemplate possible explanations of their result.

Can you think of any other possible reasons that would explain why the bottle weighs more than a kilogram?

Problem B13. How will filling the liter bottle with something other than water—for example, juice, yogurt, or sand—affect the mass? Explain. **[See Tip B13, page 63]**

Problem B14. Estimate the mass of a newborn baby, a fifth grader, an adult woman, and an adult man. If possible, use scales to gather your data. **[See Note 12]**

Note 11. If you are working in a group, when finding materials that have masses of approximately 1 g, 100 g, 500 g, and 1 kg, work individually. You can use more than one object to reach the target mass: Place the objects in a plastic bag and then label the bag with the combined mass. As a group, discuss each quantity, answering questions like "What does a mass of 1 g feel like?" and "What common items have a mass of 1 g?" Each group member should examine the different samples and then weigh them again, using the different scales.

Note 12. If you do not have a metric bathroom scale, you can use a conversion factor to change pounds to kilograms. Since 1 kg = 2.2 lb., you can find someone's weight in kilograms by dividing his or her weight in pounds by 2.2.

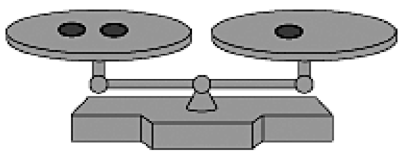
Part B, cont'd.

Reasoning With Balance Scales

Knowing something about mass and how scales respond when objects are placed on them allows us to reason logically about weight or mass. For example, fake coins sometimes make their way into circulation, and balance scales can be used to determine which coins are fake. Let's explore this further. **[See Note 13]**

Take It Further

Problem B15.



a. Imagine that you have three coins. One is heavier than normal, so you know it's a fake. But you can't tell which one is the heavy one just by looking. What is the minimum number of weighings you would need to complete to find the fake coin?

b. How would you find the fake coin among four coins? What is the minimum number of weighings needed?

c. How would you find it among six coins? What is the minimum number of weighings needed?

[See Tip B15, page 63]

d. How would you find it among eight coins? What is the minimum number of weighings needed? **[See Note 14]**

e. How would you find it among 12 coins? How many weighings would it take?

Video Segment (approximate time: 21:22–25:17): You can find this segment on the session video approximately 21 minutes and 22 seconds after the Annenberg/CPB logo. Use the video image to locate where to begin viewing.

How is the metric system used in the United States today? Watch this video segment to find out how veterinary medicine uses it to provide the best and safest treatment for its patients.

Do you know of any other professional fields that rely on the metric system?

Note 13. This activity focuses on reasoning deductively about mass, but it does not further one's knowledge of metric measures. The problems, however, do show that there are many ways to measure without using units.

Note 14. You might be tempted to solve this problem in a similar way to part (c), by first placing four coins on each of the pans. This enables you to conclude that the heavy coin is one of the four (one pan will go down!). You can then make additional comparisons in order to identify the heavy coin. There is, however, another way to solve this problem that takes only two steps. Hint: The first weighing does not involve all eight coins. Can you figure out how to identify the heavy coin?

Homework

Problem H1.

a. The meter was originally based on the size of the Earth, with the distance from the equator to the North Pole being arbitrarily defined as 10 million m. What is another way to express the distance of 10 million m?

b. The Earth is not quite spherical, but for practical purposes we can think of it as having a circumference of 40 Mm. Thus, originally the meter was considered to be about 1/40,000,000 of the Earth's circumference. Use the Web or reference books to find out how a meter is officially defined today.

Problem H2. Match the metric quantities on the left with the approximate lengths/distances on the right:

1 gigameter ($1 \cdot 10^9$)	A. distance a fast walker walks in 10 minutes
1 megameter ($1 \cdot 10^6$)	B. size of an atom
1 kilometer ($1 \cdot 10^3$)	C. waist height of an average adult
1 meter (base unit)	D. size of bacteria
1 centimeter ($1 \cdot 10^{-2}$)	E. thickness of a dime
1 millimeter ($1 \cdot 10^{-3}$)	F. distance from Atlanta to Miami
1 micrometer ($1 \cdot 10^{-6}$)	G. width of a fingernail
1 picometer ($1 \cdot 10^{-12}$)	H. Earth's distance from Saturn

Problem H3. A nickel is said to weigh 5 g. How much is 1 kg of nickels worth?

Problem H4. Give the approximate mass of the following volumes of water:

a. 6.5 L

b. 30 cm^3

c. 18 mL

d. 12 m^3

Problem H5. Why might a student be confused by this question: Which is more, 1.87 kg or 1,869 g? Explain.

Suggested Reading

This reading is available as a downloadable PDF file on the *Measurement* Web site.
Go to www.learner.org/learningmath.

The article "Do Your Students Measure Up Metrically?" points out some of the challenges of helping students in the United States learn the metric system. Discuss or think about how you might improve instruction on the metric system in your classroom or school.

Taylor, P. Mark; Simms, Ken; Kim, Ok-Kyeong; and Reys, Robert E. (January, 2001). Do Your Students Measure Up Metrically? *Teaching Children Mathematics*, pp. 282–287.

1 cm^2 Grid Paper

Tips

Part A: Metric System Basics

Tip A4. Think about the nature of the base ten system and how you can easily divide by powers of 10. Since the relationship between prefixes is based on powers of 10, we can switch prefixes easily by simply moving a decimal point.

Part B: Metric Units

Tip B9. How many cubic centimeters are there in a cubic decimeter?

Tip B10. Think about how they work and whether they measure weight or mass.

Tip B13. Think about the relationship between mass and volume. Can two substances have the same volume but different mass?

Tip B15. Rather than weighing two coins at a time, think about how you could divide the coins into groups and then compare the groups to eliminate some coins.

Solutions

Part A: Metric System Basics

Problem A1. Answers will vary. Some of the common measurements are the meter, the liter, and the gram. Most people associate the number 10 with the metric system, since relationships in this system are determined by powers of 10.

Problem A2. Most countries use the metric system because it is an international standard; it makes it easy for different countries to talk about the same units of length or mass. The metric system is also designed for easy calculation.

Problem A3.

a. There are many patterns in the table that you may want to discuss. First, the metric system uses prefixes in front of a unit name as a multiplying factor. Most prefixes multiply or divide units in steps of 1,000 as you go up and down the table.

b. The more commonly used prefixes are those from 10^{-3} to 10^{3}. These units are commonly used when measuring areas, volumes, and lengths. Throughout this session, you should focus on gaining familiarity with those particular prefixes.

c. A centimeter is 100th of a meter (1 m = 100 cm). A millimeter is 1,000th of a meter (1 m = 1,000 mm). A kilometer is 1,000 meters (1,000 m = 1 km). A micrometer is a millionth of a meter (1 m = 1,000 000 µm).

d. These measurements are related because they are all built on powers of 10, with the meter as the base unit. The prefix tells you how many meters (or how much of a meter) you're talking about.

e. A centimeter could be 10 mm or 0.01 m. A millimeter could be 0.1 cm or 0.001 m. A decimeter could be 10 cm, 100 mm, 0.1 m. Other answers are also possible!

Problem A4. One primary reason is that a number written in metric units can easily be converted by multiplying or dividing by powers of 10, since this type of multiplication is done by moving the decimal point. For example, 2.54 cm is equivalent to 25.4 mm and 0.0254 m. Each of these conversions is done by knowing the power of 10 associated with the conversion and moving the decimal point by that many places.

Problem A5.

a. As 1,000 m is 1 km, 3,600 m is 3.6 km. We simply moved the decimal point three places to the left.

b. Since 0.001 m is 1 mm, 0.028 m is 28 mm. Again, we just moved the decimal point three places to the right.

c. One way is to convert the millimeters to meters and then to kilometers: 1,000 mm equals 1 m, so 4,600,000 mm equals 4,600 m. Similarly, 1,000 m equals 1 km, so 4,600 m equals 4.6 km (i.e., we moved the decimal point six places to the left).

Problem A6. To make a fair comparison, we must first convert gigameters to megameters (or vice versa). Since 1 Gm = 1,000 Mm, the Sun's distance from the Earth is 150 • 1,000, or 150,000 Mm. The ratio is now 150,000:384, or about 390 times farther away.

Part B: Metric Units

Problem B1. Answers will vary.

Solutions, cont'd.

Problem B2.

a. Answers will vary. Distance A is approximately 1 cm. Distance B is approximately 8 or 9 cm. Distance C is approximately 1 dm. Distances D and E are approximately 1 m.

b. Answers will vary. Measure the lengths in terms of your own referent measures and then approximate.

Problem B3.

a. One way to estimate distances is to know the length of one's individual pace. First determine your average walking pace, and then use this information to approximate 100 m.

b. One way to do this is to walk the 100 m, then multiply the time it took to walk 100 m by 10. Your average walking pace would be measured in meters per second, and could be found by dividing 1,000 by the total time taken (in seconds) to walk the kilometer.

Problem B4.

a. You will need 12 metersticks to do this, one for each edge of the cube.

b. One cubic meter is equivalent to 1,000 dm^3, since there are 10 dm in each dimension and three dimensions: $10 \cdot 10 \cdot 10 = 1{,}000$. Similarly, 1 m^3 is equivalent to 1 million cm^3 and 1 billion mm^3. Since 1 L is equivalent to 1 dm^3, there are 1,000 L in 1 m^3.

Problem B5. Answers will vary, but here are a few examples:

- A teacup holds about 200 to 250 mL (or 2 to 2.5 dL).
- A thimble will probably be about a tenth of a teacup, or 20 mL.
- A gas tank of a car holds anywhere from 40 to 70 L.

Problem B6. The phrase "3 cc" refers to cubic centimeters, which is equivalent to milliliters; 3 cc of medicine is 3 mL.

Problem B7. If you measure how much liquid is in a 1 L or 2 L bottle, you'd most likely find that there is a small amount of extra liquid in each, probably as a result of a particular bottling procedure.

Problem B8. A cubic decimeter is the same volume measure as a liter. So a woman's lungs hold about 4.4 dm^3 of air, and a man's lungs about 5.8 dm^3 of air.

Problem B9.

a. The dimensions of the cubic decimeter are $10 \cdot 10 \cdot 10$ cm. The capacity is 1 L, and the weight of 1 L of water is 1 kg.

b. One cubic centimeter of water weighs 1 g. The capacity of 1 cm^3 (or cc) is 1 mL.

Problem B10. A balance scale compares two different masses. It typically only tells us when one side of the scale is larger in mass than the other.

A spring scale uses the force of gravity to determine the weight of an object placed under the spring.

The biggest difference between scale types is that some scales rely on mass calculations (typically balance scales), while others rely on weight calculations (typically spring scales). One interesting thought is whether these scales would report different answers on the Moon; the spring scale would because it depends on gravity, and the balance scale would not.

Solutions, cont'd.

Problem B11. Answers will vary. Here are some possibilities:

a. A large paper clip weighs about 1 g.
b. A chocolate bar is generally 100 g.
c. A small bag of flour is 500 g.
d. Four medium apples together will weigh approximately 1 kg.

Problem B12. Depending on the temperature of the water and the level of precision of the instrument you used, you should notice that the measured mass came close to 1 kg. This confirms what we know already: One liter of water has a mass of 1 kg when the water is at 4 degrees Celsius (4ºC).

Problem B13. Depending on the object used, the weight may not be 1 kg. For example, sand is heavier than water, so a cubic decimeter of sand (a liter of sand) should weigh more than a cubic decimeter of water. In general, a substance heavier than water but with the same volume weighs more than 1 kg. A substance lighter than water weighs less than the same volume of water.

Problem B14. A newborn baby has a mass of approximately 3 to 4 kg. A fifth grader has a mass of approximately 35 to 40 kg. An average adult woman has a mass of approximately 60 to 70 kg. An average adult male has a mass of approximately 75 to 90 kg.

Problem B15.

a. It takes only one weighing. Put one coin on one side of the balance and a second coin on the other end. If one is heavier than the other, the heavier coin is the fake. If they balance, the third coin is the fake.

b. It takes up to two weighings. Weigh two coins against each other. If one is heavier, it is the fake one. If they balance, weigh the other two coins against each other. Again, the heavier coin is the fake one.

   Alternatively, you could start by putting two coins on each side of the balance. The side with the fake coin will be heavier. Weigh those two coins against each other to determine which one is heavier, and thus fake.

c. The minimum number of weighings is still two. Put three coins on each side of the balance. The heavier side will contain the fake coin. This reduces the number of possibly heavy coins to the three on that side. Then the method from part (a) can be applied to find the heavy coin.

   Alternatively, you could divide the coins into three groups of two. Weigh two of the groups against each other. If one side is heavier, the fake coin is there. If they balance, the fake coin is in the group that was left out. Weigh the two coins in the "bad pair" against each other; the heavier is the fake one.

d. You can do this with three weighings. Put four coins on each side for the first weighing. The heavier side will contain the fake coin. This reduces the number of possibly heavy coins to the four on that side. Then divide the heavier group into two groups, and weigh them; finally, weigh the remaining two coins from the heavier group.

   You can also do this with two weighings: Place three coins on each side of the balance. If they're the same weight, proceed to weigh the two remaining coins that haven't been weighed. If not, use the method in part (a) to decide which is the fake coin.

e. It will take three weighings. First, divide the 12 coins in half, weigh them, and determine which half the heavier coin is in. Then with the heavier side, follow the six-coin strategy from part (c).

Solutions, cont'd.

Homework

Problem H1.

a. Ten million meters can also be expressed as 10,000 km, or 10 Mm.

b. A meter is defined to be the length of the path traveled by light in vacuum during a time interval of 1/299,792,458 of a second. In this way, it can be defined in terms of the second, another base unit of the metric system, and the constant speed of light.

Problem H2.

1 gigameter	H. Earth's distance from Saturn
1 megameter	F. distance from Atlanta to Miami
1 kilometer	A. distance a fast walker walks in 10 minutes
1 meter	C. waist height of an average adult
1 centimeter	G. width of a fingernail
1 millimeter	E. thickness of a dime
1 micrometer	D. size of bacteria
1 picometer	B. size of an atom

Problem H3. Since it takes 1,000 g to make a kilogram, there are about 200 nickels in a kilogram. Two hundred nickels are worth $10.

Problem H4.

a. The mass is approximately 6.5 kg.

b. Since 1 cm^3 is equivalent to 1 g, 30 cm^3 of water has a mass of 30 g.

c. Since 1 mL is equivalent to 1 g, 18 mL has a mass of 18 g.

d. Twelve cubic meters is equivalent to 12,000 dm^3. Since 1 dm^3 of water has a mass of 1 kg, 12 m^3 has a mass of 12,000 kg.

Problem H5. In terms of the number of units, 1,869 is a much larger number, so a student might be confused and say that 1,869 g is more. But since a kilogram is equivalent to 1,000 g, 1.87 kg is actually 1,870 g, which is more. This confusion might disappear once the student is more familiar with the metric system, which makes this type of conversion much easier than converting, say, inches to miles.

Notes