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How Many Blocks?

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FIFTH GRADERS BEN AN Jessica were trying to dete mine the amount of space co tained in their classroom. The chose as their unit of measure the inch-by-9-inch-by-18-inch concre block that had been used to constru the classroom walls. Because other st dents in the class had mentioned th idea, the teacher had previous brought an actual block into the clas room so that students could see a three of its dimensions. To find th number of blocks in the horizontal lay that covered the floor, they counted th blocks that occurred lengthwise in eac of two adjacent walls and multiplie They then multiplied that product b the number of blocks showing in th height of the room. For example, for small room, such as in figure 1, the would have multiplied 3 times 3, for th floor; times 5, for the height; an claimed that 45 blocks fill the room.

Even though Ben and Jessica made model of the four classroom walls b using pairs of interlocking cubes to re resent blocks, they did not detect the error—they did not see that for figure eighteen blocks fit in the layer that co ers the floor. Because these studen did not properly visualize how th blocks covered the floor, their otherwis correct procedure of multiplying th number of blocks in a horizontal lay by the number of layers produced an i correct answer.

Spatial Structuring

THE DIFFICULTY THAT BEN AND JESSICA HAD with this problem is caused by their improper spatial structuring. To spatially structure an object or set of objects is to construct mentally an organization or form for it by identifying its component parts and how they are spatially related. To determine the number of blocks that fit in the room in **figure 1**, we must spatially structure the set of blocks, that is, we must imagine exactly how to organize the blocks so that they completely fill the room.

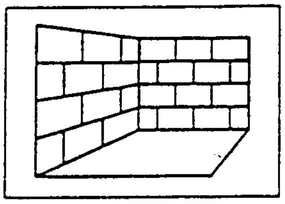


Fig. 1 Concrete blocks in walls

Spatial structuring plays a crucial role in geometric reasoning. For example, research shows that students' spatial structuring determines how they compute the number of cubes in a three-dimensional array of cubes (Battista and Clements 1996). Students who spatially structure an array into columns or layers generally calculate the total by skip counting or multiplying by the number of cubes in a column or layer. Alternatively, many students structure an array as an unrelated set of rectangular-prism faces. They determine the number of cubes visible on all or some of these faces, usually counting cubes along the prism's edges more than once (see **fig. 2**). In fact, about 40 percent of fifth through eighth graders use this structuring-deficient type of strategy (Ben-Chaim, Lappan, and Houang 1985).

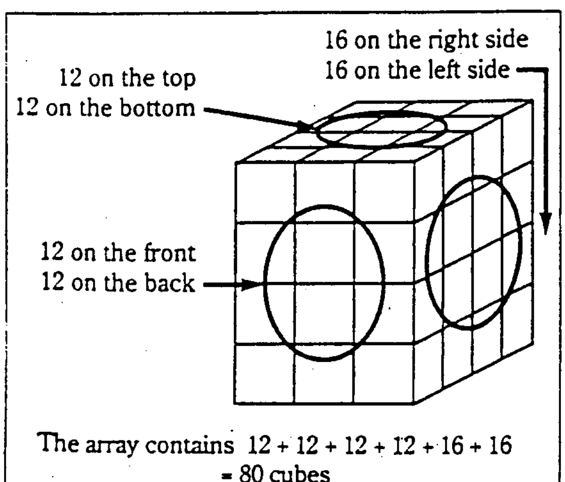


Fig. 2 A common student strategy for enumerating cubes in an array

This article discusses the extent of students' difficulties with spatial structuring in volume and packing problems. It also describes how to help students develop more powerful ways of thinking about such problems.

Similar Problems, Similar Results

TO SEE HOW OTHER STUDENTS WOULD DEAL with the type of situation faced by Ben and Jessica, a similar problem was given to forty-five fifth graders (see **fig. 3**). Of the twenty-eight students who answered incorrectly, twenty-three did so because they made errors similar to those errors made by Ben and Jessica. For instance, some students multiplied 5 by 3 to get the number of packages in a horizontal layer, then multiplied by four layers. Other students multiplied 5 by 4 to get twenty packages in a vertical layer along the left side, then multiplied by 3. Still other students counted twelve packages along the back, then multiplied by 5.

When checking their answers with paper boxes and packages made from interlocking cubes, many students still had difficulties determining how the packages fit into the box. For instance, Anita asserted that five packages fit into the box across the length and three across the width. To test her assertion, she placed three packages along the width. But after placing four packages along the length,

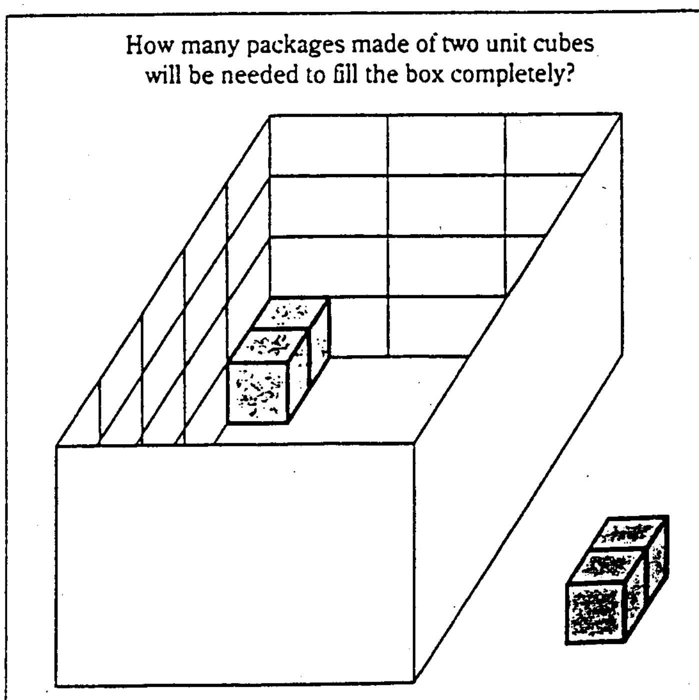


Fig. 3 A packing problem

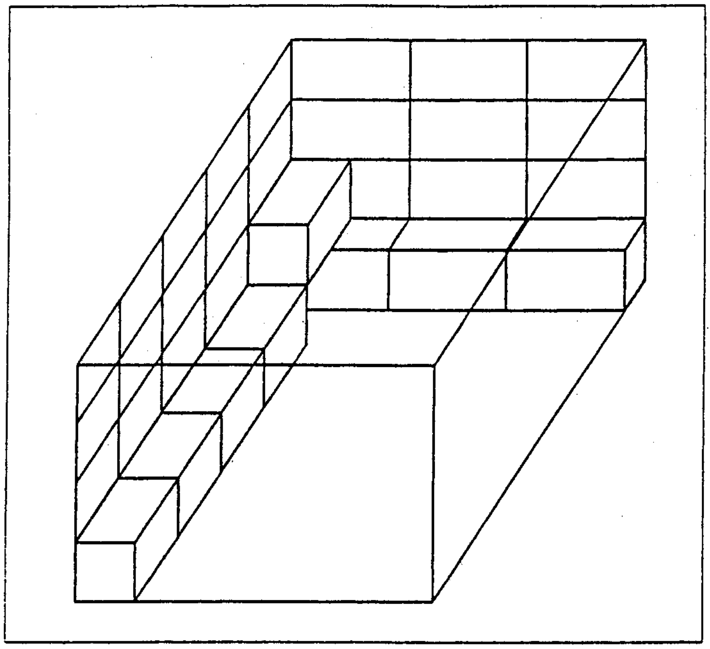


Fig. 4 Anita's placement of packages

she was perplexed that the fifth package would not fit into the bottom of the box while the three packages along the width remained (see **fig. 4**). Eventually, Anita decided that she could place the packages into the box, all with their longest side parallel to the longest side of the box. But she was unable to determine the number of packages that covered the bottom of the box until she completely covered it in this way.

To investigate students' structuring difficulties further, a group of teachers of grades 3 through 5 administered an even simpler task to their students (see **fig. 5**). **Table 1** reveals that a majority of students made the same structuring error as did Anita and her classmates, answering 15.

Students making the "15" error gave explanations and drawings clearly indicating that they had structured the box as either three columns of five packages or five rows of three packages (see **fig. 6**).

Student 1: We know 3 will fit across; we know will fit downward; just multiply.

Student 2: I kept counting 5 along the side of th package until I counted 3 times because you ca only fit 3 packages along the top of the box.

Other students used visual estimation that i nored portions of the given information. For i stance, one student drew the picture in **figure** and answered 20, stating that the box containe four columns of five packages. Still other studen focused on the perimeter of the box. One studer for example, answered 16, explaining, "I put 5 (both sides and 3 on the other sides and added."

Each of these students enumerated packages I using inadequate spatial structuring. In particul the structuring of students who answered seemed especially superficial. Apparently, tl salience of the illustrations that five packages along one side of the box and three along the oth automatically activated students' use of the famil procedure "there's three groups with five in each. multiply 3 times 5." Using this procedure caus them to bypass a careful structural analysis of h the packages could fill the box.

The fact that the "15" error increased as gr level increased suggests that the more autom the multiplication procedure became for stude the less likely it was for them to use it appropria in this nonroutine situation. This suggestion is f ther supported by the student responses that f low. When presented with a problem similar to t one shown in **figure 3**, except with a box of heig 3, the students responded as follows.

John: [John, a seventh grader, was three wee from completing a standard course in high sch algebra.] 45. You just use the volume formula, ba times height times width.

TABLE 1
Percent of Students Giving Each Type of Answer for the Packing Problem in Figure 5

Answer	Grade 3 (80 Students)	Grade 4 (93 Students)	Grade 5 (117 Students)
Correct	4	10	9
Incorrect			
15	41	53	64
Perimeter	8	9	3
Visual estimation (answer ≠ 15)	19	8	10
Other	29	22	13

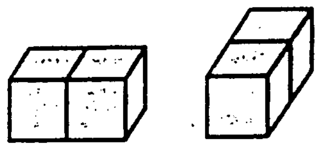
Note: Because the percents have been rounded, the resulting sums do not total 100.

Fig. 5
Problem given to students in grades 3–5.

All students were given the problem sheet below. The teacher then read the problem to the students while showing how an actual package made from two interlocking cubes fit into a paper box, five along the length and three along the width. Students solved the problem and then gave a written description of how they solved it.

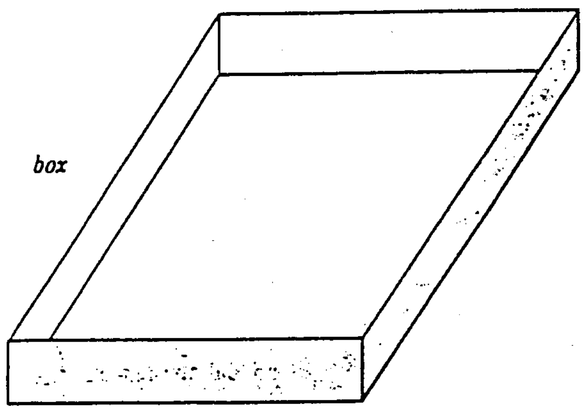
Problem Sheet

Jill is making packages that each contain two cubes.

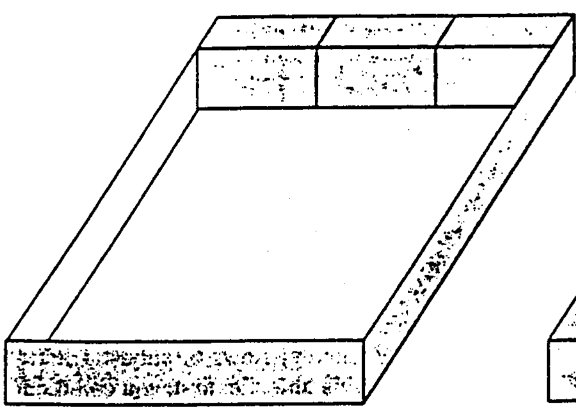


packages

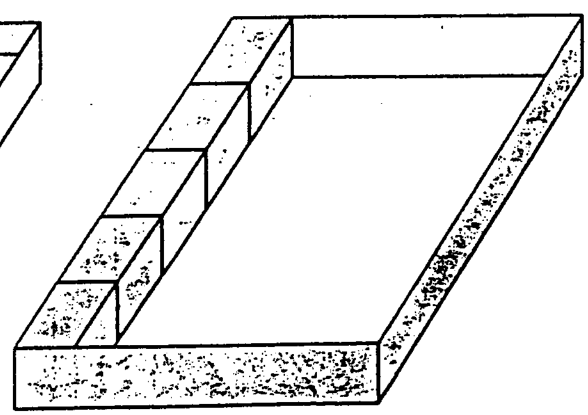
Jill wants to know how many of these packages fit into the rectangular box shown below.



She knows that she can fit three packages along the top of the box.



She knows that she can fit five packages along the side of the box.



How man packages can Jill fit into the box? _______

Observer: How did you use this formula?

John: 3 times 3 times 5. You can fit 3 for the height, 3 along the width, and then 5 along the length [pointing to each one of these dimensions].

Clarissa: [Clarissa, an eighth grader, was three weeks from completing a standard course in high school geometry.] It's 45 packages. And the way I found it is I multiplied how many packages could fit in the height by the number in the width, which is 3 times 3 equals 9. Then I took that and multiplied it