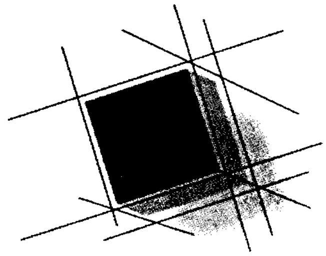
Perspective

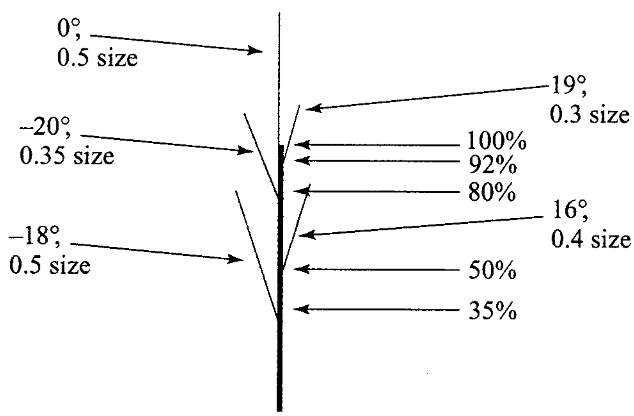


What Is Geometry?

Geometry is the attempt to understand space, shape, and dimension. Parts of "geometry"—earth-measuring—grew out of the age of explorers to map where they had been, and of landowners to determine the boundaries of their holdings. Other parts were invented by artists, who wished to portray convincingly what they saw with their eyes or saw in their minds, and by inventors and engineers who wished to make devices that would fit together and work. Geometrical ideas have also come from the needs of architects and builders whose work needs to be both strong and beautiful, and from surveyors, planners, and workers who must be able to assure that tunnels or railroad tracks built from both ends will actually meet in the middle.

Geometric shapes are not only the simple shapes with familiar names like *square* or *pyramid.* Fractal geometry can describe tree-like shapes with surprisingly simple methods.

The following picture describes the rule for building the trees as shown after Problem **1.**



1. Decide how to interpret this pictorial "rule."

a. How many limbs does it show growing from the trunk?

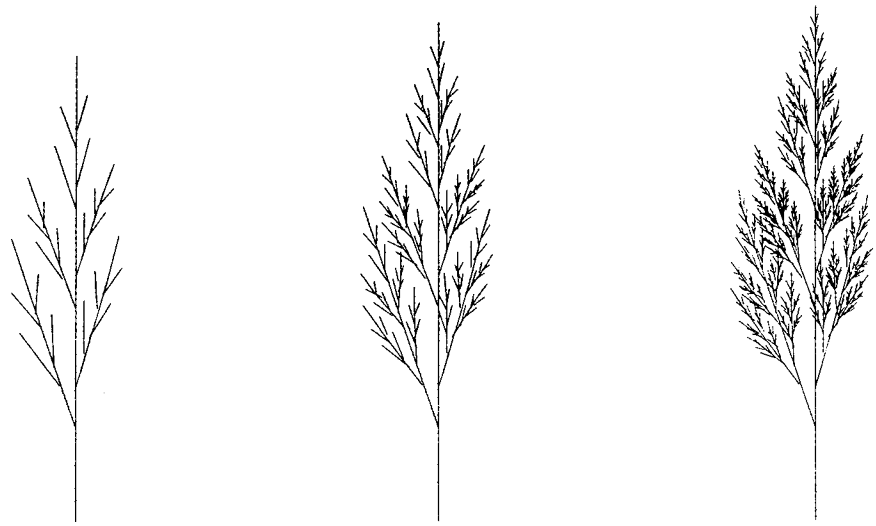
b. How far up the trunk is the lowest limb?

c. How long is that limb compared with the trunk?

d. What angle does that limb make with the trunk?

e. How far up the trunk is the highest limb?

f. How do its size and angle compare with those of the trunk?



The picture on the left shows what happens when each limb sprouts five branches according to the same rules—the same relative distance along the branch, size relationships, and angle of growth. Apply the same rules again to grow five twigs on each branch; and the result looks like the middle picture. The last picture, with five leaves per twig, is very tree-like.

Mathematics is sometimes pictured as if it were a field unto itself, unrelated to other classes, fields, or hobbies. And geometry is often presented as unrelated even to other kinds of mathematics. Historically, none of this was the case. Within mathematics, geometry has been a source of great insight into other mathematical domains, like algebra, and vice versa. Some of the most important mathematicians were also artists, scientists, inventors, clerics, or a blend of these careers. Art, mathematics, science, and social sciences are still closely related. One of the fastest growing scientific fields combines the study of mathematics, psychology, computer science, and biology. Artist M.C. Escher (1898–1972) was fascinated with mathematical ideas and incorporated many into his drawings. Dancer and choreographer Michael Moschen uses ideas from mathematics and physics to inspire his remarkable art.

Examples abound of mathematics influencing other fields and vice versa, but these are often neglected when subjects are taught separately. The *Connected Geometry* units are designed to help you find these connections or invent them for yourself, building links with your other interests and studies, and within mathematics itself.

***Euclid's series of books was titled* The Elements.**

How has geometry changed in the last 3000 years?

The most famous geometry books of all time, *The Elements,* were written in Egypt about 2300 years ago. Their author, Euclid, compiled and systematized in them his own ideas and all the geometrical ideas that he had known, many of which had been developed by the mathematicians who lived before him.

But mathematics, including geometry, is not all thousands of years old, all known, or all finished with nothing new to discover. It should be no surprise that a lot of new geometry has been invented since Euclid. After all, 2300 years is a long time. Art, architecture, mechanical design, clothing design, navigation, communication, and other fields have all changed and have raised new geometrical questions.

In fact, geometry has changed enormously in the last twenty years, and it continues to change rapidly today. New applications, especially involving computers, have changed the scope of what is possible to explore in geometry.

Computer graphics and animation have created new jobs and demanded new research. New geometrical techniques have been developed to solve problems of optimizing paths—for example, finding the most efficient routes for snow plows, the best routes for airlines, the least expensive network of phone wires, or the smallest microchip. The mathematical study of knots, which started in support of a defunct theory held by chemists a century ago, has recently become a powerful tool in modern physics and biochemistry. Astrophysicists have new notions of the shape of space and have invented new mathematical tools like optical geometry. The advent of the computer age has made possible mathematical modeling of complex natural phenomena like weather, and has given rise to a new branch of mathematics called dynamical systems. And there is combinatorial geometry, algebraic geometry, differential geometry, and so on. This is one reason why such different problems as the ones you have just done can all be considered geometry.

Education Development Center, Inc. (2000). Perspective: What Is Geometry? In *Connected Geometry: A Habits of Mind Approach to Geometry.* Glencoe/McGraw-Hill. <www.glencoe.com/sec/math>