

Annenberg/CPB Course Guide

Learning Math

Patterns, Functions, and Algebra

A 10-part video- and Web-based course for K-8 teachers

Produced by WGBH Educational Foundation

Learning Math: Patterns, Functions, and Algebra

is produced by
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About This Course

The *Learning Math* Courses

When teachers have a deep conceptual understanding of mathematics, it can help their students develop strong mathematical skills and knowledge. *Learning Math* is a series of five multimedia, college-level courses designed to teach mathematics content to elementary and middle school teachers. Organized around the content standards developed by the National Council of Teachers of Mathematics (NCTM), the courses will cover Number and Operations; Patterns, Functions, and Algebra; Geometry; Measurement; and Data Analysis, Statistics, and Probability.

Overall Objectives

- To help teachers better understand mathematics content
- To provide engaging explorations of mathematics using video, interactive activities, and problem solving
- To encourage teachers to view mathematics as more than rote sets of rules and procedures

Overview: *Patterns, Functions, and Algebra*

Patterns, Functions, and Algebra explores the “big ideas” in algebraic thinking, such as finding, describing, and using patterns; using functions to make predictions; understanding linearity and proportional reasoning; understanding nonlinear functions; and understanding and exploring algebraic structure. The course consists of 10 two-and-a-half hour sessions that each include video programming and activities, provided in this guide and on the Web. The 10th session explores ways to apply the algebraic concepts you’ve learned in K-8 classrooms. You should complete the sessions sequentially.

Session 1. Algebraic Thinking

Discover what it means to think algebraically, and learn to use algebraic thinking skills to make sense of different situations. Topics covered include describing situations through pictures, charts, graphs, and words; interpreting and drawing conclusions from graphs; and creating graphs to match written descriptions of real-life situations.

Session 2. Patterns in Context

Explore the processes of finding, describing, explaining, and predicting using patterns. Topics covered include how to determine if patterns in tables are uniquely described and how to distinguish between closed and recursive descriptions. This session also introduces the idea that there are many different conceptions of algebra.

Session 3. Functions and Algorithms

Investigate algorithms and functions. Topics covered include the importance of doing and undoing in mathematics, determining when a process can or cannot be undone, using function machines to picture and undo algorithms, and recognizing that functions produce unique outputs.

Session 4. Proportional Reasoning

Look at direct variation and proportional reasoning. This investigation will help differentiate between relative and absolute meanings of “more” and compare ratios without using common denominator algorithms. Topics include differentiating between additive and multiplicative processes and their effects on scale and proportionality, and interpreting graphs that represent proportional relationships or direct variation.

Session 5. Linear Functions and Slope

Explore linear relationships by looking at lines and slopes. Using computer spreadsheets, examine dynamic dependence and linear relationships and learn to recognize linear relationships expressed in tables, equations, and graphs. Also, explore the role of slope and dependent and independent variables in graphs of linear relationships, and the relationship of rates to slopes and equations.

About This Course, cont'd.

Session 6. Solving Equations

Look at different strategies for solving equations. Topics include the different meanings attributed to the equal sign and the strengths and limitations of different models for solving equations. Explore the connection between equality and balance, and practice solving equations by balancing, working backwards, and inverting operations.

Session 7. Nonlinear Functions

Continue exploring functions and relationships with two types of nonlinear functions: exponential and quadratic functions. This session reveals that exponential functions are expressed in constant ratios between successive outputs and that quadratic functions have constant second differences. Work with graphs of exponential and quadratic functions, and explore exponential and quadratic functions in real-life situations.

Session 8. More Nonlinear Functions

Investigate more nonlinear functions, focusing on cyclic and reciprocal functions. Become familiar with inverse proportions and cyclic functions, develop an understanding of cyclic functions as repeating outputs, work with graphs, and explore contexts in which inverse proportions and cyclic functions arise. Explore situations in which more than one function may fit a particular set of data.

Session 9. Algebraic Structure

Take a closer look at “algebraic structure” by examining the properties and processes of functions. Explore important concepts in the study of algebraic structure, discover new algebraic structures, and solve equations in these new structures.

Session 10. Classroom Case Studies

Explore how the concepts developed in *Patterns, Functions, and Algebra* can be applied at different grade levels. Using video case studies, observe what teachers do to develop students’ algebraic thinking and investigate ways to incorporate algebra into K-8 mathematics curricula. This session is divided into three grade bands: K-2, 3-5, and 6-8.

Course Components

Each *Learning Math* course consists of 10 two-and-a-half-hour sessions. The first nine sessions are devoted to mathematics content; the 10th session covers classroom applications. Concepts are developed within and across the sessions and the sessions increase in difficulty as they progress. Each session includes reading, problem solving, and group or individual activities that are available on the Web and in print, and a half-hour of video viewing, available on the Web,* on the Annenberg/CPB Channel,** or on videocassette.*** There are additional problems and readings to complete for homework.

* Broadband access is required to view the video on the Web; see Tech Tips, page 5.

** The schedule for broadcasts on the Annenberg/CPB Channel can be found on the course Web site.

*** Purchase videocassettes at www.learner.org or by calling 1-800-LEARNER.

Visit the *Patterns, Functions, and Algebra* Web site at www.learner.org/learningmath.

About This Course, cont'd.

The following components are in each course:

Key Terms

Key mathematical terms relevant to each session are listed at the beginning of that session. These terms are divided into two parts: terms that are new in that session and terms that were introduced in a previous session. Definitions for key terms may be found in the glossary in the Appendix of this guide.

Notes

Notes can be used by facilitators, study groups, or individuals working alone. They provide extended information about the topics presented in the course, including help for dealing with stumbling blocks that may come up and recommendations for different ways to approach the content.

Problems

Each session contains mathematical problems to be solved individually or by groups. Problems build upon previous concepts and increase in difficulty as the course progresses.

Take It Further

The problems marked "Take It Further" are optional and are not counted as part of the two-and-a-half hour time-frame for each session. These problems are designed for individuals who would like to explore a topic in greater depth. They are often more difficult than the other problems in the session, and they may introduce new information or concepts not previously discussed.

Interactive Activities

Each session in the course includes at least one interactive activity on the course Web site. These activities help you learn new mathematics content or reinforce existing knowledge through hands-on exploration directly on the Web. The interactive activities require the Flash plug-in, which you can download for free from Macromedia's Web site (see Tech Tips, page 5). There are also non-Flash versions of each activity that don't require the Flash plug-in and can be completed offline. If you are working with only the guide, the interactive activities have been adjusted and are included in the guide.

Tips

Tips are available for problems you may find more difficult or need help in getting started. Tips may be found at the end of each session in this guide.

Solutions

A solution is provided for every problem in *Learning Math*, with the exception of a few open-ended questions. When solving a problem with multiple parts, consider writing down your answers to all of the parts on paper first before checking the solution, because the answers to each part of the problem will be visible at once on the solution page. Solutions may be found at the end of each session in this guide.

The following sequence of activities will give you a sense of what you will do as a student using *Learning Math*:

1. Watch the **session video** in its entirety. You can watch the video before you begin the session to become comfortable with the material, or you can view the video after completing the session (so as not to view answers to problems).
2. Do problems sequentially. If you are having difficulty, refer to the **Tip**. If you want a challenge, try a **Take It Further**.
3. Check the **Solutions** at the end of each session.
4. You may want to read **Notes** as you go along to establish a deeper context for the content.
5. Watch **video segments** strategically placed throughout the session, either online by clicking on the "Play Video" button or on videocassette by fast-forwarding to the image and approximate time code that appears in the guide. Zero your VCR clock when the Annenberg/CPB logo appears at the beginning of the program to locate the image using the time code.
6. Do **homework problems** and **readings** (available as PDF files online) at the end of each session to reinforce your learning.

About This Course, cont'd.

Session Videos

Each *Learning Math* session includes viewing a video that is available on the course Web site, on videotape, or on the Annenberg/CPB Channel. The videos feature K-8 teachers working on the *Learning Math* course materials in a workshop with a facilitator. The videos for the nine content sessions show onscreen participants as they are introduced to the concepts, work through the problems, sometimes struggle to reach an understanding, and then reflect on what they have learned. At the end of most videos there is an example of how the content from the session is applied in a “real world” situation. The videos for the 10th session show participants from the videotaped workshops as they apply the content that they have learned back in their own classrooms. You may choose to watch each of these videos before or after you work on the associated course session.

Video Segments

Each session includes short excerpts from the associated video, which you watch (or review) and reflect on to see how the onscreen participants grapple with the same or similar problems and concepts you encounter in the course. Instructions are given to find the segments on videotape. The segments are also available on the course Web site, if you are watching the complete programs online or on the Annenberg/CPB Channel.

Homework

Each session includes approximately 45 minutes of homework problems and/or reflective writing assignments that reinforce the session’s content.

Readings

Readings from journals and books are cited at the end of some sessions. They are available on the *Patterns, Functions, and Algebra* Web site as downloadable PDF files. Go to www.learner.org/learningmath. If you do not have access to the Internet, call 1-800-LEARNER to obtain a free set of readings.

Using the Videos, Guide, and Web Site

Each *Learning Math* course includes sequentially organized problems, video viewing, interactive activities, readings, and homework. The multimedia elements of the course create an exciting environment for probing mathematics content. The course can be taken entirely on the Web, followed in this print guide, or completed using a combination of Web and print. You can watch the videos online, on the Annenberg/CPB Channel, or on videocassette. If you are watching the programs on the Channel, we recommend taping them so you can look at short video segments when prompted in each session. These segments punctuate concepts developed in the course and create a “virtual” community of learners.

Ways To Take *Learning Math*

Learning Math was flexibly designed for a variety of users and situations. You may choose to work through the sessions on your own, in a study group, or as part of a facilitated, face-to-face, graduate-level course for credit.

Channel-Talk

Join an email discussion group and converse with other teachers taking this course. Go to the course Web site at www.learner.org/learningmath and select Channel-Talk.

Registration and Credit

Go to www.learner.org/4gradcredit for details on receiving graduate credit for *Learning Math*.

About This Course, cont'd.

Taking Multiple *Learning Math* Courses

The five *Learning Math* courses are designed to be independent of one another. You can take just one course, several courses, or all five courses in the order that fits your needs or the needs of your group. The courses also complement one another, with some topics discussed in more than one course but approached differently depending on the focus of that course. Taking several of these courses will increase your own conceptual understanding and ultimately that of your students.

Facilitating the Course

You can prepare for facilitating the course by reading each session and its accompanying "Notes" prior to meeting with your group. The Notes suggest ways to organize groups for discussions, problem solving, and activities. Specific suggestions for "Groups" within the Notes should be useful to group facilitators or members of a self-directed study group.

Tech Tips

To use all or portions of the *Learning Math* course online, we recommend the following:

Internet Access

A minimum 56K modem connection is required, but an ISDN or high-speed connection is recommended. The slower your connection, the longer it will take to load larger features, such as the Flash activities.

To view the video programs and video segments online, a broadband connection to the Internet (DSL, cable modem, or LAN connection to a T1 line or greater) is required.

Web Browser

You will need Netscape 4.6 (or a higher version) or Internet Explorer 5.0 (or a higher version). Javascript should be enabled, if your browser allows you to disable it. Text fonts and colors, and many features, such as hidden Tips, may not be displayed correctly in older browsers.

Plug-Ins

These plug-ins will allow you to get the most out of the courses. Links to sites where you can download these programs for free are provided on the course Web site.

- Shockwave Flash 5 (or a higher version) for using the Interactive Activities
- Windows Media Player 7 (or a higher version) for watching videos
- Adobe Acrobat Reader for viewing the Readings in the Homework sections

Printing Web Pages

If you are having trouble printing some of the course content pages, you may try doing one or more of the following (from the "Print Preview," "Print...," or "Page Setup..." menu):

- Turn on "Shrink to Fit" mode (IE 5 only)
- Print the page in "Landscape" mode
- Reduce the scale of the printer output

Who's Who

Course Instructor



Ruth Cossey,
Mills College, Calif.

Onscreen Participants



Sue-Anne Brabant
K-1 Teacher/Coordinator



Celeste Janey
Special Education Teacher,
Grades K-2



Liza Jones
Teacher, Grade 3



Laurie Levin
Instructional Leader in
Elementary Mathematics



Lolita Mattos
Math Teacher, Grades 7-8



Ric Mosher
Math Teacher, Grades 7-9

Who's Who, cont'd.



Blake Munro
Elementary Math Coordinator



Christina Orr
Bilingual Teacher, Grade 3



Frederick Won Park
Teacher, Grades 3-4



Andrea Plate
Math Teacher, Grade 8



Deanna Purdy
Math Teacher, Grade 9



Mary Lou Randall
Math Teacher, Grade 7



John Souto
Math Teacher, Grades 7-8



Hazel Sparrow
English/Language Arts Teacher,
Grades 7-8



Bette Toney
Teacher, Grade 1



Gina Webber
Teacher, Grade 1

